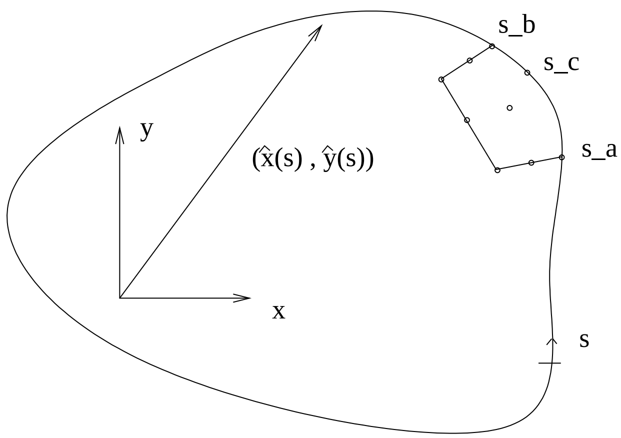
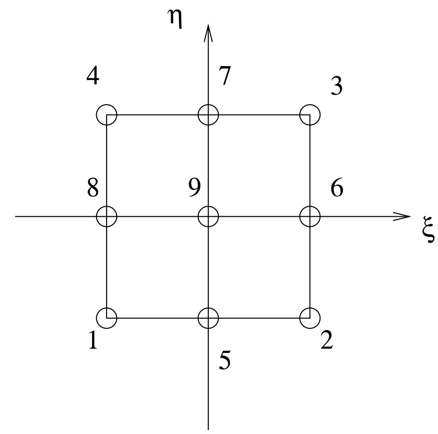
Surface Loads





Consider a 2D problem with a boundary integral,

$$I = \int_\Gamma f(s) ds \tag{1}$$

where $s = s_a$ gives the arc length location of the first node on the element via the parametric function $(\hat{x}(s_a), \hat{y}(s_a))$ defining the boundary. The end node on the boundary segment is at $s = s_b$ and thus at $(\hat{x}(s_b), \hat{y}(s_b))$. The mid-side node on the boundary segment is at $s = s_c$ and thus at $(\hat{x}(s_c), \hat{y}(s_c))$. Let us assume that the correspondence to the parent domain is that this edge is mapped to parent domain nodes $3, 4, 7$, respectively.

To compute I, we can exploit the isoparametric map $x(\xi, \eta), y(\xi, \eta)$ to effect a change of variables from $s \in (s_a, s_b)$ to $\xi \in (-1, 1)$. The primary consideration is that

$$\hat{x}(s_a) = x(\xi_3, 1) \qquad \hat{y}(s_a) = y(\xi_3, 1) \tag{2}$$

$$\hat{x}(s_b) = x(\xi_4, 1) \qquad \hat{y}(s_b) = y(\xi_4, 1) \tag{3}$$

$$\hat{x}(s_c) = x(\xi_7, 1) \qquad \hat{y}(s_c) = y(\xi_7, 1)\,. \tag{4}$$

The mapping between the s-domain and the ξ-domain that will respect these relations is simply

$$s = s_a N_3(\xi, 1) + s_b N_4(\xi, 1) + s_c N_7(\xi, 1)\,. \tag{5}$$

In this case

$$I = \int_{s_a}^{s_b} f(\hat{x}(s), \hat{y}(s)) ds \tag{6}$$

$$= \int_1^{-1} f(x(\xi, 1), y(\xi, 1)) \frac{ds}{d\xi} d\xi\,. \tag{7}$$

In this case the Jacobian of the transformation is negative which is compensated for by the inverted limits of integration – positive to negative. Alternately one can write

$$I = \int_{-1}^{1} f(x(\xi, 1), y(\xi, 1)) \left| \frac{ds}{d\xi} \right| d\xi \,, \tag{8}$$

which works in all cases. Notwithstanding, most FEA programs will set up a local arclength coordinate on the element edge and map $s_a \to -1$, $s_b \to 1$, $s_c \to 0$. Thus the Jacobian is always positive. It should also be noted using as common result from dynamics of point-masses, that

$$\left| \frac{ds}{d\xi} \right| = \sqrt{\left[\frac{dx}{d\xi}(\xi, 1) \right]^2 + \left[\frac{dy}{d\xi}(\xi, 1) \right]^2} \,. \tag{9}$$

Thus one never needs to fix an origin for the arclength origin nor does one ever have to explicitly compute arclength positions of points on the surface.