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Tutorial exercises on forced oscillations

Some of you have not studied forced oscillations of linear systems. If you feel uncomfortable with the topic here is a set of exercises that you can perform to help get a better feel for the topic (all in 1D with scalar variables). To gain anything from these exercises you need to work them out in detail and think about the results.

First consider following differential equation of motion for a spring-mass system:

$$m\ddot{u} + ku = f \,, \tag{1}$$

where m is the mass, k the stiffness, u the displacement, and f the load.

1. First assume that $f = \hat{f}e^{i\omega t}$ (i.e., harmonic) and that $u = \hat{u}e^{i\omega t}$, where ω is a fixed driving frequency for the load. Show that

$$\hat{u} = \frac{\hat{f}}{k - \omega^2 m} \,. \tag{2}$$

2. Make a plot of $\hat{u}k/\hat{f}$ the (nondimensionalized) system response versus ω/ω_o, where $\omega_o = \sqrt{k/m}$. Plot your result over the range $\omega/\omega_o \in [0, 2]$. Note that ω_o is the natural frequency of vibration of the system. Think about the meaning of the plot. What does it tell you?

3. Consider now a system with linear viscous/structural damping governed by a damping constant c. In this case the governing equation for the spring-mass-damper system is

$$m\ddot{u} + c\dot{u} + ku = f \,. \tag{3}$$

Assume again that the forcing is harmonic with frequency ω and the response is also harmonic with the same frequency. Show that the response can now be written as

$$\hat{u} = \frac{\hat{f}}{k + i\omega c - \omega^2 m} \,. \tag{4}$$

4. Nondimensionalize your result from the previous step to show that

$$\frac{\hat{u}k}{\hat{f}} = \frac{1 - \omega^2/\omega_o^2 - i\beta\omega/\omega_o}{(1 - \omega^2/\omega_o^2)^2 + \beta^2\omega^2/\omega_o^2} \,, \tag{5}$$

where $\beta = \omega_o c/k$. Hint: To show this multiply the numerator and denominator of the result form the previous step by $(k - \omega^2 m) - i\omega c$.

5. Plot the magnitude of the nondimensionalized response from your last result versus ω/ω_o for $\beta \in \{0.1, 0.3, 0.7, 1.0, 2.0\}$. Make the plot on a log-log scale over the range $\omega/\omega_o \in [0, 2]$. Interpret your plot. What does it mean? Hint: In MATLAB you can compute the magnitude of a complex number with the command `abs`.

6. Over the same range plot the phase angle of the nondimensionalized response on a semi-log plot (log on the nondimensional frequency axis, linear for the phase angle). What does the plot say? Hint: In MATLAB you can compute the phase angle of a complex number with the command `angle`.

7. Consider the forcing $f = \hat{f}\sin(\omega t)$ and find the steady state solution directly from the governing ODE (ignore any transient terms). How is this solution related to the solution for $\hat{u}$ that you have already computed? Hint: $\hat{u}$ has both imaginary and real parts.

8. Consider the forcing $f = \hat{f}\cos(\omega t)$ and find the steady state solution directly from the governing ODE (ignore any transient terms). How is this solution related to the solution for $\hat{u}$ that you have already computed? Hint: $\hat{u}$ has both imaginary and real parts.

9. Consider the forcing $f = \hat{f}\sin(\omega t + \phi)$ (where ϕ is a given constant) and find the steady state solution directly from the governing ODE (ignore any transient terms). How is this solution related to the solution for $\hat{u}$ that you have already computed? Hint: $f = \text{Im}\left\{\hat{f}e^{i\phi}e^{i\omega t}\right\}$.