HW 8: Due Thursday April 18

1. (50pts total)

   An important question in conducting a mechanical analysis with a time varying load is to decide if the problem needs to be solved by computing the solution to the full dynamical equations (including the inertial terms)

   $$\boldsymbol{M}\ddot{\boldsymbol{u}}(t) + \boldsymbol{K}\boldsymbol{u}(t) = \boldsymbol{F}(t)$$

   or can one reasonably ignore the inertial terms and solve the simpler relations

   $$\boldsymbol{K}\boldsymbol{u}(t) = \boldsymbol{F}(t)\,,$$

   the so-called quasi-static approximation.

   (a) (25 pts) Using the concept of converting the mechanical equations into a set of uncoupled second order ordinary differential equations, develop a criteria to decide this question.

   (b) (25 pts) Consider now the problem from HW 7 but now subjected to a force at $x = L = 1$ m, where the force as a function of time is given by

   $$F(t) = F_o \begin{cases} 0 & t < 0 \\ t/t_r & 0 \le t < t_t \\ 1 & t \ge t_r\,, \end{cases}$$

   where F_o is the magnitude of the load and t_r is the rise time of the load. Apply your criteria from Question (1a) to determine for which (range of) values of t_r one needs to solve the problem using the full dynamical equations and when can one get away with a quasi-static approximation. Demonstrate your answer with illustrative numerical examples.