HW 7: Due Thursday March 21

1. (50pt total) Consider a one-dimensional elastic bar whose dynamic behavior is governed by

$$AEu'' = A\rho\ddot{u} \tag{1}$$

Assume that the bar is fixed at $x = 0$ and is free at $x = L$. With these boundary conditions the exact natural frequencies of vibration are given by

$$\omega_j^{\text{exact}} = \sqrt{\frac{E}{\rho}}\frac{1}{L}\left(\frac{\pi}{2} + j\pi\right) \qquad j \in \{0, 1, 2, \ldots\} \tag{2}$$

(a) (20 pts) Find and plot the error in the finite element approximation for ω_0 and ω_3 when using linear as well as quadratic elements. The relative unsigned error $|\omega_j^{\text{fea}} - \omega_j^{\text{exact}}|/\omega_j^{\text{exact}}$ should be plotted versus L/d, where d is the nodal spacing. Use log-log scaling. The range of L/d should be sufficient for the relative unsigned error to drop below 10^{-6} for both vibrational modes. All your data should appear on a single plot.

Assume $L = 1$ m, $A = 0.01$ m^2, $E = 200$ GPa, and $\rho = 5000$ kg/m^3.

[Hint: Always sort the output of MATLAB's eigensolvers. You should observe that linear elements converge more slowly than the quadratic elements and the higher modes also converge slower than the lower modes.]

(b) (10 pts) Plot the mode shapes associated with the first and third modes of vibration.

(c) (20 pts) Assume initial conditions $u(x, 0) = 10^{-3}\frac{x}{L}$, $\dot{u}(x, 0) = 0$ and compute the transient response of the bar by converting to 1st order form and time integrating the solution using Backward Euler from $t = 0$ to $t = 1.5$ ms. Compare the solution when using linear versus quadratic elements and different choices for Δt. Note this is somewhat open ended. Try to do something that you thinks highlights the numerical behavior of the solution you are computing.