HW 5: Due Thursday Feb. 28

1. (10 pts) Consider 3-node triangular element with nodes located at

$$\boldsymbol{x}_1^e = (0, 0) \text{ mm} \quad (1)$$
$$\boldsymbol{x}_2^e = (1, 1) \text{ mm} \quad (2)$$
$$\boldsymbol{x}_3^e = (-1, 1) \text{ mm}\,. \quad (3)$$

Assume that the nodal temperatures for this element have been computed to be $T_1^e =$ 100 K, $T_2^e = 150$ K, $T_3^e = 300$ K.

   (a) Assuming $T(\boldsymbol{x}) = \boldsymbol{N}^e \boldsymbol{T}^e$, what is the temperature at the point $(0, 0.5)$ mm?

   (b) Compute the heat flux vector in the element assume that the material conductivity is isotropic with $k = 150$ W/m · K.

   (c) Compute the heat flux vector in the element assume that the material conductivity is anisotropic with

$$\boldsymbol{D} = \begin{bmatrix} 150 & 25 \\ 25 & 100 \end{bmatrix} \text{ W/m} \cdot \text{K}\,.$$

2. (10 pts) Consider a 9-node bi-quadratic isoparametric quadrilateral element.

   (a) Construct an expression for the shape function associated with the (corner) node located at $(-1, -1)$ in the parent domain.

   (b) Make a plot of this shape function in the parent domain, verifying that your expression in Part 2a is indeed one at the node located at $(-1, -1)$ and zero at all the other nodes.

3. (10 pts) Consider the problem of species diffusion where the governing physical principal is mass conservation. Starting with this concept applied to an arbitrary region $\mathcal{R}$, derive as we did for heat conduction the governing partial differential equation in the steady state case; see Table 6.1 in Ottosen and Petersson. With that in hand, derive the weak form and construct expressions for the FE matrix equations.