UNIVERSITY OF CALIFORNIA BERKELEY   Structural Engineering,
Department of Civil Engineering   Mechanics and Materials
Spring 2019   Professor: S. Govindjee

HW 4: Due Thursday Feb. 21

1. (10 pts) Consider a single 1-D 4 node Lagrangian element with nodes located at $x_1^e, x_2^e, x_3^e, x_4^e$.

   (a) Write expressions for the Lagrange shape functions of this element.

   (b) What is the polynomial order of approximation for the element? i.e. what order polynomial variation can this element exactly represent.

   (c) Plot your shape functions over a single element (assuming equally spaced nodes).

   (d) Consider the following mesh. Construct the LM array for it using the given number scheme

Element 1   Element 2   Element 3
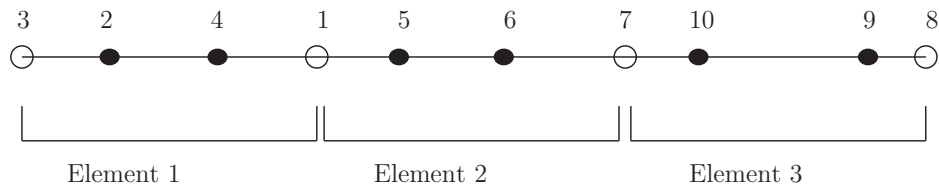
   (e) The element stiffness matrix, $\boldsymbol{k}^e$, for such an element is a 4×4 matrix. Into which location (row and column) of the global stiffness matrix will the element stiffness component $k_{12}^{e=3}$ be assembled?

2. (10 pts) The 1-D 4-node cubic *isoparametric* element has nodes in the parent domain located at $-1, -1/3, 1/3, 1$.

   (a) Write expressions for the isoparametric shape functions $N_i(\xi)$.

   (b) Assuming an element with physical nodes located at x equals $0, 1, 2$ and 4, write out the isoparametric mapping, $x^e(\xi)$. (You do not need to expand the shape function expressions.)

   (c) Find an expression for the isoparametric Jacobian $j^e(\xi) = \partial x^e / \partial \xi$.

3. (30 pts) Consider the following problem

$$u'' + x^{-3/2} = 0 \qquad x \in (0.1, 2) \tag{1}$$

subject to boundary conditions $u'(0.1) = 0$ and $u(2) = 0$. The exact solution to this problem is

$$u_{\text{exact}}(x) = 4\sqrt{2}(-1 + \sqrt{5}) + 4\sqrt{x} - 2\sqrt{10}x. \tag{2}$$

Compute a finite element solution to this problem and using plots compare $u_{\text{exact}}(x)$ to $u_{\text{fea}}(x)$ and compare $u'_{\text{exact}}(x)$ to $u'_{\text{fea}}(x)$. You should compute two different finite element solutions:

(a) using a single 3-node isoparametric element with nodes located at 0.1, 1.05, and 2.0 – i.e. equally spaced nodes.

(b) using a single 3-node isoparametric element with nodes located at 0.1, 0.8, and 2.0 – i.e. *non*-equally spaced nodes.

This problem is to be solved by writing a MATLAB (or Python) program. This program has been started for you `hw4p3_student.m` (`hw4p3_student.py`). Read through the file, locate all instances of the text *[FINISH]*, and fill in the missing code. Do not change the names of the variables that have already been defined for you.

Make sure to turn in your code along with your plots.

Start early in order to be able to complete on time.