HW 2 (corrected 2/9/19 15:50): Due Thursday Feb. 14

1. Consider the following system of equations

$$\begin{bmatrix} 1 & -1 & & & & & & \\ -1 & 2 & -1 & & & & & \\ & -1 & 2 & -1 & & & & \\ & & -1 & 2 & -1 & & & \\ & & & -1 & 2 & -1 & & \\ & & & & -1 & 2 & -1 & \\ & & & & & -1 & 1 & \end{bmatrix} \begin{pmatrix} a_1 \\ 3 \\ a_3 \\ a_4 \\ a_5 \\ 7 \\ a_7 \end{pmatrix} = \begin{pmatrix} 1 \\ f_3 \\ 3 \\ 4 \\ 5 \\ f_7 \\ 7 \end{pmatrix}$$

   Solve for the unknown a_i and f_i.

2. Let the location matrix for a given problem be given by

$$\text{LM} = \begin{array}{|c|c|c|c|c|c|} \hline 1 & 3 & 2 & 6 & 5 & 4 \\ \hline 2 & 4 & 3 & 7 & 6 & 5 \\ \hline \end{array}$$

   Assume for all elements e that the local element stiffness and force vectors are given by

$$\boldsymbol{k}^e = \begin{bmatrix} 50 & -50 \\ -50 & 50 \end{bmatrix} \qquad \boldsymbol{f}^e = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

   and assemble the global stiffness and force vectors.

3. A thin heart shaped shaped domain as shown in Fig. 1 is discretized by 4 (3-node) elements. The element stiffness matrix for elements 1 and 2 is given by

$$\boldsymbol{k}^e = \begin{bmatrix} 40 & -50 & 5 \\ -50 & 100 & -50 \\ 5 & -50 & 40 \end{bmatrix} \tag{1}$$

   and for elements 3 and 4 it is given by

$$\boldsymbol{k}^e = \begin{bmatrix} 50 & -60 & 15 \\ -60 & 200 & -60 \\ 15 & -60 & 50 \end{bmatrix} \tag{2}$$

   (a) Construct a location matrix LM for this geometry and numbering scheme.

   (b) Assemble the global stiffness matrix.

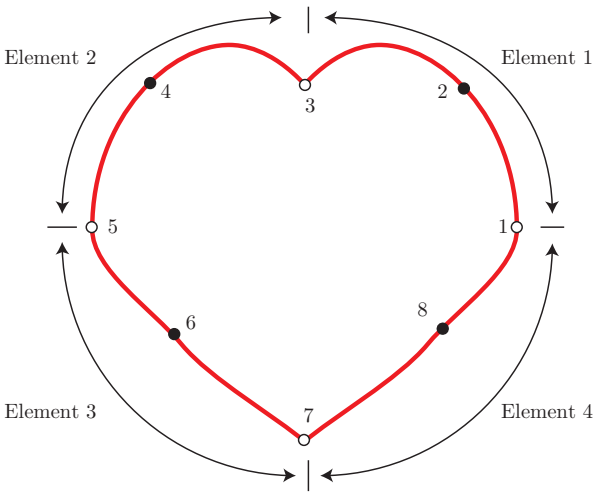

Figure 1: Heart shaped domain.