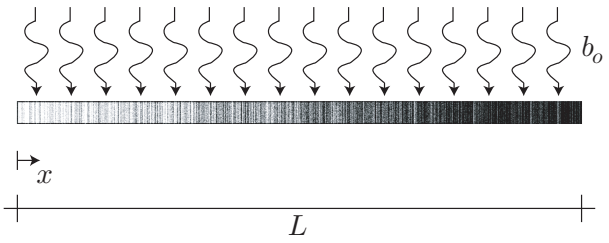
**HW 1: Due Thurday Jan. 31**

1. A low-molecular weight substance $\gamma$ is being injected into a *one-dimensional* reservoir at a constant rate $b_o$ ($\mathrm{g/m^3 \cdot s}$). $\gamma$ is being consumed by a chemical reaction at a spatially inhomogeneous rate $\alpha(x) = \alpha_1 + \alpha_2 \frac{x}{L}$ (1/s), where $\alpha_1, \alpha_2$ are given constants. $\gamma$ also diffuses laterally with a spatially inhomogeneous diffusivity $D(x) = D_1 + D_2 \frac{x}{L}$ ($\mathrm{m^2/s}$), where $D_1, D_2$ are given constants. Derive an expression for the governing balance law for $\gamma$. [Hint: Conservation of mass.]




2. Consider the (strong form) problem of finding $u(x)$ such that

$$\frac{d}{dx}\left[\frac{du}{dx}\right] + 2 = 0 \tag{1}$$

$$u(L) = u_L \tag{2}$$

$$\frac{du}{dx}(0) = F_o \tag{3}$$

   Derive the weak form (W) of the governing balance equation? and provide a statement of the weak form problem (WP).

3. Consider the following weak form problem: Find $u(x) \in \mathcal{S}$ such that

$$\int_0^1 w'u' + wu\,dx = 5w(1) \tag{4}$$

   for all $w \in \mathcal{V}$, where $\mathcal{S} = \{u(x) \mid u(0) = 1\}$ and $\mathcal{V} = \{w(x) \mid w(0) = 0\}$.

   (a) What is the governing second order differential equation that matches this weak form — i.e. what is the associated strong form?

   (b) Show that the expression on the left hand side represents a symmetric bi-linear form.

   (c) What is the minimization form (M) associated with this weak form?