UNIVERSITY OF CALIFORNIA BERKELEY
Department of Civil Engineering
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Structural Engineering,
Mechanics and Materials
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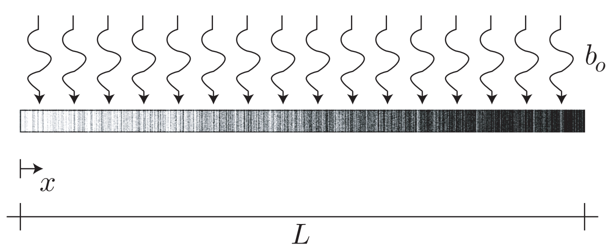
HW 1: Due Thurday Feb. 4

1. A cylindrical water tank of radius R has a water level of $h(t)$. Water flows into the tank at a rate of $q_{in}(t)$ (m^3/day) and is drawn out at a rate of $q_{\mathrm{out}}(t)$ (m^3/day). Use the conservation of mass (assume water to be incompressible) to arrive at the governing equation for $h(z,t)$. [Hint: Should be a first order differential equation for $h(t)$.

2. A low-molecular weight substance γ is being injected into a *one-dimensional* reservoir at a constant rate b_o ($\mathrm{g/m^3 \cdot s}$). γ is being consumed by a chemical reaction at a spatially inhomogeneous rate $\alpha(x) = \alpha_1 + \alpha_2 \frac{x}{L}$ (1/s), where α_1, α_2 are given constants. γ also diffuses laterally with a spatially inhomogeneous diffusivity $D(x) = D_1 + D_2 \frac{x}{L}$ ($\mathrm{m^2/s}$), where D_1, D_2 are given constants. Derive an expression for the governing balance law for γ. [Hint: Conservation of mass.]



3. Consider the (strong form) problem of finding $u(x)$ such that

$$\frac{d}{dx}\left[(1+x)\frac{du}{dx}\right] + 2 = 0 \tag{1}$$

$$u(L) = u_L \tag{2}$$

$$\frac{du}{dx}(0) = F_o \tag{3}$$

   (a) What is the space of trial solutions for this problem?
   (b) What is the space of (admissible) test functions for this problem?
   (c) What is the weak form of the governing balance equation?

4. Consider the following weak form problem: Find $u(x) \in \mathcal{S}$ such that

$$\int_0^1 w'u' + wu \, dx = 5w(1) \tag{4}$$

for all $w \in \mathcal{V}$, where $\mathcal{S} = \{u(x) \mid u(0) = 1\}$ and $\mathcal{V} = \{w(x) \mid w(0) = 0\}$.

Find an approximate Galerkin solution using the approximations $\mathcal{S}_2 = \{u(x) \mid u(x) = u_1 x + u_2 x^2 + 1\} \subset \mathcal{S}$ and $\mathcal{V}_2 = \{w(x) \mid w(x) = w_1 x + w_2 x^2\} \subset \mathcal{V}$. Compare your solution to the exact solution. [Hint: What is the governing second order differential equation that matches this weak form?]