## Lab 6

In many engineering contexts there is a desire to have estimates of the effective properties of composite systems. Consider for example the composite wall shown in Fig. 1. The wall is composed of four different materials with the left-side exposed to a high temperature and the right-side to a low temperature. The temperature induce a flow of energy from left to right which varies spatially in a complex manner. If one is interested in overall thermal design using such wall elements, what is of primary importance is knowledge of the effective conductivity of the wall, defined as:

$$\bar{k} = \frac{Q}{\Delta T} \tag{1}$$

where Q is the total heat flowing though the wall and  $\Delta T$  is the temperature difference across the wall; the units (SI) of  $\bar{k}$  are [W/K].



Figure 1: Wall unit cross-section. Conductivities are given in units of  $[W/m \cdot K]$ . Lengths are in cm. Top and bottom faces are fully insulated.

For the wall system shown in Fig. 1

1. Use a thermal circuit analysis to determine the interior temperatures between the wood faces and the interior elements; see Fig. 2. The effective conductivities in the circuit are given by  $\bar{k}_{(.)} = k_{(.)}A_{(.)}/t_{(.)}$ , while assuming a unit thickness into the page. For example,  $\bar{k}_c = 0.76 \cdot (0.5 \cdot 1)/0.075 = 5.06\bar{6} [W/K]$ .



Figure 2: Thermal circuit for wall unit.

- 2. Compare these temperature values to the FEA results. Note the FEA computation will give you T(y) on the interior faces of the wood elements whereas as the circuit analysis will give single values.
- 3. Determine the percentage of x-direction heat that flows through the concrete versus that which flows through the glass wool in the circuit analysis and the FE analysis. Compare.
- 4. Compare the sum of the x-direction heat that flows through the concrete and glass wool to that which flows through the front face of the fir.
- 5. Lastly find  $\bar{k}$  for the wall unit from the circuit analysis and the FE analysis. Compare.