University of California, Berkeley
CEE C133/ME C180, Engineering Analysis Using the Finite Element Method
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Lab 3: Relation between Mesh Refinement and Higher Order Approximations and Convergence

This week, we will be studying the effects of mesh refinement on the convergnce of physical properties that we don't necessarily know the exact solution for, and how different order interpolation functions effect convergence.

1. Start up COMSOL as follows:
   - **Structural Mechanics** module
   - **Plane Stress/Static analysis**
2. Create the geometry as follows:
   - Rectangle, with $0 \leq x \leq 10\,m$ and $0 \leq y \leq 2\,m$
   - Circle, with center at $(x, y) = (5, 1)\,m$, and $r = 0.5\,m$
   - Use the **Difference** operator to make a circular hole in the plate.
3. Set the material properties and boundary conditions as follows:
   - Youngs modulus equals $7 \times 10^{10} \frac{N}{m^2}$
   - Poissons ratio equals 0.33
   - Note that we will be doing these tests for both **Lagrange - Linear** and **Lagrange - Quadratic** elements, so be sure you have the correct option chosen.
   - Left end: Select the correct boundary condition which enforces $u_x = 0$, and u_y unconstrained.
   - Right end: Add a uniformly distributed load with a resultant of $1000N$ in the x direction, and uniformly equal to $0N$ in the y direction.
4. Define the mesh size as described below, under **Mesh/Free Mesh Parameters**
5. Solve the problem.

Your assignment is as follows:

1. Iterate the **Maximum element size** over the values 1.0, 0.8, 0.6, 0.4, 0.2, 0.1, 0.09, 0.08, 0.07, 0.06, 0.05. For each mesh, write down the maximum value of von Mises stress.

2. Do the preceeding twice, once for **Lagrange - Linear** elements, and once for **Lagrange - Quadratic** elements.

3. Plot (in MATLAB, Excel, or whatever else you want) the stress concentration factor K versus the element size h (both cases on the same plot), where the stress concentration factor is defined as

$$K = \frac{\sigma_{max}(h)}{\sigma},$$

   where $\sigma_{max}(h)$ is the maximum von Mises stress for each different value of element size, and σ is the von Mises stress for the case of no hole in the bar. As the color bar on the right of the screen may sometimes show values that are not the absolute maximum of a quantity over the domain, take the maximum von Mises stress from the **Max/Min** plot type.

   Recalling that reducing the element size is referred to as *h-refinement*, and increasing the order of the interpolation functions is referred to as *p-refinement*, comment on which type of refinement appears more useful in this case, and justify your comments. (I am thinking this should be no more than two or three sentences.)

4. For any case you want, plot the following:

   - Surface plot, von Mises stress
   - Arrow plot, on Boundaries, Surface traction vector
   - Deformed shape plot, auto scale factor

Turn in both of the plots, your comments on the effects of the different types of refinement, and your data.