University of California, Berkeley
CEE C133/ME C180, Engineering Analysis Using the Finite Element Method
Spring 2010
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Lab 2: Mesh Refinement and Error

This week, we will be studying the effects of mesh refinement, and how it effects the convergence of the FE solution. Consider the strong form of our canonical problem:

$$A\frac{\partial}{\partial x}\left(E(x)\frac{\partial u}{\partial x}\right) + b = 0, \text{ in } \Omega = (0, L),$$
$$u = \bar{u}, \text{ on } \Gamma_u = \{0\},$$
$$AE\frac{\partial u}{\partial x} = \bar{F}, \text{ on } \Gamma_q = \{L\}.$$

Note that this looks a bit different than the form the Professor Govindjee has typically written in class; indeed, this form is more general, and corresponds to the one shown in class, when E is a constant. The assignment is as follows:

1. Calculate the exact solution of the PDE, including determination of the constants of integration. Assume A and b are constants, and $E = E_1(1 + x)$, with E_1 constant. Show your work.

2. Model the canonical problem, as follows:

   (a) In the Model Navigator, select **1D**, **PDE Coefficient Form**, **Stationary analysis**, and **Lagrange - Linear** elements.

   (b) Create your geometry.

   (c) Use the following data:

   - $A = 1 \times 10^{-4}$
   - $E = E_1(1 + x)$, where $E_1 = 7 \times 10^{10}$
   - $\Omega = (0, 1)$
   - $\bar{u} = 0$
   - $b = 100$
   - $\bar{F} = 100$

   (d) Enter all of the appropriate terms into the **Subdomain Settings** and **Boundary Settings** dialog boxes; take particular note to use the correct signs.

3. Run the model for **Free Mesh Parameters/Maximum element size** as follows: 1, 0.5, 0.33, 0.25, 0.15, 0.1, 0.05, 0.01. Be sure to hit **Initialize Mesh** after each time you change the mesh size.

4. Save the solutions for the different mesh densities in the files *mesh1.txt*, *mesh2.txt*, ..., *mesh8.txt*. This can be done by choosing **File/Export/Postprocessing Data** from the main menu, and setting **Node Points for Lagrange Elements of Order** equal to 1, and choosing **Format of Exported Data** to be "Coordinates, Data".

5. Download and run the script *me180_lab2_convergence.m* from bspace. This will create a plot of error versus element size.

6. Hand in the derivation of the analytical solution to the PDE, as well as the error plot.