UNIVERSITY OF CALIFORNIA BERKELEY
Department of Civil Engineering
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Structural Engineering,
Mechanics and Materials
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HW 5: Due Thursday March 4

1. **The isoparametric concept 1-D:** In standard Lagrangian elements the unknown field over a single element is given by $\phi(x) = \sum_{i=1}^{n} \phi_i^e \psi_i^e(x)$, where $\psi_i^e(x)$ is a polynomial equal to 1 at x_i^e and zero at the other nodes of the element. In the isoparametric setting $\phi(\xi) = \sum_{i=1}^{n} \phi_i^e \psi_i^e(\xi)$, where $x = \sum_{i=1}^{n} x_i^e \psi_i^e(\xi)$ and the shape functions are given as polynomials over the parent domain and equal to 1 at the associated node and zero at the other nodes.

   Compare the behavior of a quadratic Lagrange element with a quadratic isoparametric element by plotting the representation of $\phi^e(x)$ over a single element as assumed in each formulation. Consider a situation where $\phi_1^e = 1$, $\phi_2^e = .5$, and $\phi_3^e = 1.5$ and where the physical nodes are

   (a) located at $x_1^e = 0.0$, $x_2^e = 2.0$, $x_3^e = 4.0$,

   (b) located at $x_1^e = 0.0$, $x_2^e = 2.5$, $x_3^e = 4.0$.

   In both cases plot ϕ^e over the physical element – i.e. over the domain $(0, 4)$. In the case of the Lagrange element this is very direct but in the case of the isoparametric element you will need to invert the isoparametric map since one easily knows ϕ^e only over the parent domain. Remark: In the first case you should see that both element formulations give the same interpolation. In the second case you should see that they differ. A numerical solution (to 3 digits of accuracy) for the inversion of the isoparametric map is sufficient, if you do not want to do it by hand.

2. JNR 9.13. For this problem he asks that you compute the Jacobian. You should also make plots of the Jacobian determinant over the element for the cases of $a = 0$, $a = -0.25$, $a = -0.5$, $a = -0.75$. Axes are labeled wrong in the books figure. They should be x,y not ξ, η.

3. How many nodes would one need in a bi-cubic quadrilateral element?

4. Check if the isoparametric shape functions for a 4 node quadrilateral element are linearly complete (recall this was a requirement for convergence of isoparametric finite element methods for problems governed by second order differential equations); i.e. check that the sum of the shape functions is identically unity at all points.