UNIVERSITY OF CALIFORNIA BERKELEY
Department of Civil Engineering
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Structural Engineering,
Mechanics and Materials
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HW 2: Due Thursday Feb. 11

1. Formulate the weak form problem for the following strong form statement: Find $\phi(x)$ such that
$$\phi'' + 1 = 0 \qquad \forall x \in (0,1)\,,$$
where $\phi(0) = \phi(1) = 0$. Find an approximate (Bubnov-Galerkin) finite element solution using finite element function spaces composed of a single linear hat function centered at $x = 0.5$. Plot your solution against the exact solution.

2. Consider the functional
$$\Pi(\phi) = \int_0^1 [(\phi')^2 + x\phi]\,dx + 5\phi(1)\,.$$
Minimize this functional over the space of functions $\mathcal{S} = \{\phi \mid \phi(0) = 1\}$. Leave your result in the weak form.

3. Consider the following problem
$$\phi'' + \sin(\pi x) = 0$$
over the domain $x \in (0,1)$ with boundary conditions $\phi(0) = \phi(1) = 1$. Assume the following approximate trial function space
$$\mathcal{S}_N = \{\phi_N(x) \mid \phi_N(x) = 1 + \sum_{j=1}^{3} \phi_j \sin(\pi j x)\}\,.$$
   (a) Show that the exact solution to this problem in an element of $\mathcal{S}_N$.
   (b) Using the standard Bubnov-Galerkin prescription determine values for the ϕ_js. Your test function space will be:
$$\mathcal{V}_N = \{w_N(x) \mid w_N(x) = \sum_{j=1}^{3} w_j \sin(\pi j x)\}\,.$$
   You do not need to do this by hand, it is also ok to do it numerically. Integrals can be computed using quad4 in Matlab, for example.
   (c) Comment on the result you have obtained in part (b) relative to part (a).

4. Consider $u'' + b = 0$ over a domain $(0, \pi)$. Assume boundary conditions $u(0) = 0$ and $u'(\pi) = 1$, and a distributed load $b(x) = \delta(x - \pi/2)$. For a space of trial solutions assume:

$$\mathcal{S}_N = \{u_N(x) \mid u_N(x) = \sum_{i=1}^{N} u_i x^i\}$$

and assume a similar expression for $\mathcal{V}_N$.

(a) Show that the linear equations governing the unknowns u_i have the form

$$\sum_{i=1}^{N} K_{ji} u_i = F_j \, ,$$

where

$$K_{ji} = \frac{ij}{i+j-1} \pi^{(i+j-1)}$$

and

$$F_j = \pi^j + (\pi/2)^j \, .$$

(b) Using Matlab, solve these equations for $N \in \{1, 2, 3, 4, 5, 10, 50, 100, 200, 300, 400, 500\}$. (Note that you may not get solutions for the higher values of N depending upon your computer.)

i. Plot your solutions against the exact solution.

ii. Plot the error $e_N(x) = u_{\text{exact}}(x) - u_N(x)$ for each value of N.

iii. Plot the derivative error $e'_N(x) = u'_{\text{exact}}(x) - u'_N(x)$ for each value of N.

iv. Plot the log of the L^2 norm of the error versus the log of N; i.e. make a plot of the L^2 norm of the error versus N on a log-log plot. When computing this norm, it is ok to use a discrete sum if you want instead of exactly computing the norm.

In this problem you will have a lot of plots. Organize the presentation of your results in a clear fashion. Do not forget to properly label the axes; use legends and titles on all plots or figure captions. Do not use a single page for each plot; you should probably put at least 4 to 6 plots on each page. You need to also make some comments on the behavior you see. Think and comment. Make a professional looking presentation of your results. Turning in a raw stack of plots will not get you any credit for this problem.

Useful definitions:

1. L^2 inner product of two functions over the interval (a, b)

$$\langle f, g \rangle = \int_a^b f(x) g(x) \, dx$$

2. L^2 norm over the interval (a, b)

$$\|f\| = \sqrt{\langle f, f \rangle} = \sqrt{\int_a^b f^2 \, dx}$$

3. Discrete L^2 norm is simply a Riemann sum approximation to the integral in the above definition.