University of California, Berkeley
CEE C133/ME C180, Engineering Analysis Using the Finite Element Method
Spring 2009
Instructor: S. Govindjee
GSI: N. Hodge

**Lab 2**

This week, we will be studying the effects of mesh refinement, and how it effects the convergence of the FE solution. Consider the strong form of our canonical problem:

$$\begin{aligned} A\left(E(x)u'\right)' + b = 0, &\quad \text{in } \Omega = (0, L), \\ u = \bar{u}, &\quad \text{on } \Gamma_u = \{0\}, \\ AEu' = \bar{F}, &\quad \text{on } \Gamma_q = \{L\}. \end{aligned}$$

Note that this looks a bit different than the form the Professor Govindjee has typically written in class; indeed, this form is more general, and corresponds to the one shown in class, when $E$ is a constant. The assignment is as follows:

1. Calculate the exact solution of the PDE, including determination of the constants of integration. Assume $A$ and $b$ are constants, and $E = E_1(1 + x)$, with $E_1$ constant.
2. Model the canonical problem, as follows:
   (a) In the Model Navigator, select **1D**, **PDE Coefficient Form**, **Stationary analysis**, and **Lagrange - Linear** elements.
   (b) Create your geometry.
   (c) Use the following data:
   - $A = 1 \times 10^{-4}$
   - $E = E_1(1 + x)$, where $E_1 = 7 \times 10^{10}$
   - $\Omega = (0, 1)$
   - $\bar{u} = 0$
   - $b = 100$
   - $\bar{F} = 100$

   (d) Enter all of the appropriate terms into the **Subdomain Settings** and **Boundary Settings** dialog boxes; take particular note to use the correct signs.
3. Run the model for **Free Mesh Parameters/Maximum element size** as follows: 1, 0.5, 0.33, 0.25, 0.15, 0.1, 0.05, 0.01. Be sure to hit **Initialize Mesh** after each time you change the mesh size.

4. Save the solutions for the different mesh densities in the files *mesh1.txt*, *mesh2.txt*, ..., *mesh8.txt*. This can be done by choosing **File/Export/Postprocessing Data** from the main menu, and setting **Node Points for Lagrange Elements of Order** equal to 1, and choosing **Format of Exported Data** to be "Coordinates, Data".

5. Download and run the script *me180_lab2_convergence.m* from bspace. This will create a plot of error versus element size.

6. Write your name and the analytical solution to the PDE on your plot, and turn it in.