UNIVERSITY OF CALIFORNIA BERKELEY
Department of Civil Engineering
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Structural Engineering,
Mechanics and Materials
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HW 9: Due April 16

1. Consider the following system of equations which result after the spatial discretization of a one dimensional system with finite elements:

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{K}\boldsymbol{u} = \boldsymbol{F}\,,$$

where

$$\boldsymbol{M} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \text{(kg)} \tag{1}$$

$$\boldsymbol{K} = \begin{bmatrix} 4 & -3 \\ -3 & 3 \end{bmatrix} \text{(N/m)} \tag{2}$$

and

$$\boldsymbol{F} = \begin{pmatrix} F_1(t) \\ F_2(t) \end{pmatrix} \text{(N)}\,,$$

where

$$F_1(t) = \begin{cases} 0 & t < 0 \\ t & 0 \leq t < 1 \\ 1 - (1 - t) & 1 \leq t < 2 \\ 0 & 2 \leq t\,, \end{cases}$$

$$F_2(t) = \begin{cases} 0 & t < 0 \\ 2t & 0 \leq t < 1/2 \\ 1 & 1/2 \leq t\,. \end{cases}$$

Assume that for all times $t \leq 0$ that the system is quiescent. Compute the system response over the time interval $[0, 100]$ (s) using the following schemes.

(a) Backward Euler

(b) Forward Euler

(c) ODE45 in MATLAB

(d) Newmark's method $\gamma = 1/2$, $\beta = 1/4$

(e) Newmark's method $\gamma = 0.9$, $\beta = 0.45$

(f) Newmark's method $\gamma = 1/2$, $\beta = 0$ (careful with the β)

Comment on the results and the nuances of the various schemes.