HW 6: Due March 19

1. Consider a domain $\Omega = \{\boldsymbol{x} \mid 0 < x < a \text{ and } 0 < y < b\}$ and steady-state heat conduction

$$\kappa \nabla^2 T + r(x,y) = 0 \qquad \forall \boldsymbol{x} \in \Omega\,,$$

where $r(x,y) = \kappa T_1 4\pi^2 \frac{4b^2+a^2}{a^2b^2} \sin(4\pi x/a)\cos(2\pi y/b)$, $T(0,y) = 0$, $T(a,y) = T_2 y/b$, $T(x,0) = T_1 \sin(4\pi x/a)$, and $T(x,b) = T_1 \sin(4\pi x/a) + T_2 x/a$. The exact solution to this problem is

$$T(x,y) = T_1 \sin\left(\frac{4\pi x}{a}\right)\cos\left(\frac{2\pi y}{b}\right) + T_2 \frac{xy}{ab}\,.$$

   (a) Verify that the given solution is the exact solution.

   (b) Assume that $a = 30$ cm, $b = 10$ cm, $\kappa = 109$ W/mK, $T_1 = 40$ C, and $T_2 = 40$ C. Using linear triangular elements find the solution to this problem. Approximately how many elements are need to find a solution with (relative) error less than 1% in the L^2 norm.

   (c) Redo part (1b) using quadratic triangular elements.

   (d) Using quadratic triangular elements, determine the total normal heat flux along the edge $x = a$ to an accuracy of 3 digits; i.e. find $\int_0^b \boldsymbol{q}(a,y)\cdot\boldsymbol{e}_x dy$ such that your answer is correct in its first 3 digits. Approximately how many elements were needed?

   Note that there are a number of ways that you can post process the data. Within COMSOL you can directly manipulate the results and compute functions of the solution. When doing this, note that T is temperature, x x-position, y y-position, nflux_ht is normal heat flux. Also under the menu option File/Export/Postprocessing Data you can export data to a file. Best is to check the box that says *write script that creates postprocessing data*. This then creates a matlab script file that you can run. The output can be controlled to just give coordinates and data or to also give element information. Under the subdomain button you can also export not just temperature but also any function you can type in the dialog-box. Be careful with boundary data if you decide to export it. I suggest using some very simple problems to try and decipher how it exports this information. Don't forget about the help manual.