HW 5: Due March 5

1. Consider a two dimensional domain $\Omega = \{\boldsymbol{x} \mid \|\boldsymbol{x}\| < R\}$, where $R = 1$ m. Let the domain be composed of a Portland cement with thermal conductivity $k = 0.29$ W/mK. Assume a temperature field $T(x, y) = T_o + T_d\sqrt{x^2 + y^2}/R$, where $T_o = 20$ C and $T_d = 100$ C.

   (a) Find and plot the heat flux field over the domain.

   (b) What is the total energy per unit time moving through the boundary of the domain? Is it moving in or out of the domain?

   (c) Is this field a steady state temperature distribution for a system with $r = 0$? If not, what must r equal for steady state? Specify and plot if not zero.

2. Write out explicitly the thermal stiffness matrix for a 3 node Lagrangian triangle for a material with isotropic heat conductivity k. Assume that the three nodes are located at $\boldsymbol{x}_1^e = (0, 0)$, $\boldsymbol{x}_2^e = (1, 1)$, $\boldsymbol{x}_3^e = (0, 1)$.

3. Assume in Problem 2 that over the edge between nodes 1 and 3 that there is a constant specified outward heat flux $\bar{q}$. Compute the entries of the nodal heat flux vector for the element.

4. Assume in Problem 2 that over the edge between nodes 1 and 3 that there is a specified outward heat flux $\bar{q}_o + \bar{q}_1 \frac{y}{L}$, where $L = 1$ and $\bar{q}_o, \bar{q}_1$ are given constants. Compute the entries of the nodal heat flux vector for the element.

5. Assume that in Problem 2 that there is a constant distributed volumetric heat input $r = r_o$. Compute the entries of the nodal heat flux vector for the element.

6. Compute an analytic solution to the following boundary value problem. Let $\Omega = \{\boldsymbol{x} \mid 0 < x < a \text{ and } 0 < y < b\}$.

$$\nabla^2 T = 0 \qquad \forall \boldsymbol{x} \in \Omega\,,$$

   where $T(x, 0) = T_1$, $T(x, b) = T_2$, $T(0, y) = 0$, $T(a, y) = T_2$.