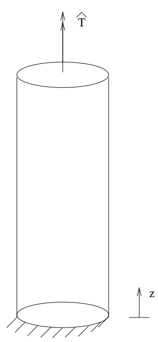
UNIVERSITY OF CALIFORNIA BERKELEY — Structural Engineering,
Department of Civil Engineering — Mechanics and Materials
Spring 2009 — Professor: S. Govindjee

HW 4: Due Thursday Feb 26

1. Consider a functionally graded torsion bar as shown; the potential energy is given by

$$\Pi(\phi(z)) = \int_0^L \frac{1}{2} G(z) J (d\phi/dz)^2 \, dz - \hat{T}\phi(L) \,, \tag{1}$$

   where J is the polar moment of inertia, $G(z) = G_1 + G_2 z$ is a spatially varying shear modulus, and ϕ is the section rotation.



   Assume a mesh composed of two linear finite elements of equal length and determine the solution $\phi^h \in \mathcal{S}^h$ which minimizes Π. Compare to the exact solution $\phi(z) = \frac{\hat{T}}{JG_2} \ln(1 + \frac{G_2}{G_1} z)$ by plotting both solutions. In your plot, normalize the rotation ϕ by $\hat{T}/JG_2$, position along the bar by L, and assume that $G_2 L / G_1 = 2$. Note: This problem is to be done by hand with exact integrations.

2. Consider a single 1-D 4 node Lagrangian element with nodes located at x_1, x_2, x_3, x_4.

   (a) What is the order of the element? i.e. what is the degree of completeness?

   (b) Write expressions for the shape functions of this element.

   (c) Plot your shape functions.

   (d) Consider a 1-D bar, discretized by 3 of these elements. Create a global numbering scheme for the mesh and write out the LM matrix.

3. The 1-D 4 node *isoparametric* element has nodes in the parent domain located at $-1, -1/3, 1/3, 1$.

   (a) Write out the isoparametric shape functions $N_a^e(\xi)$.

(b) Assuming an element with nodes located at $0, 1, 1.5, 2$. Write out the isoparametric mapping.

(c) Compute the isoparametric Jacobian $\partial x/\partial \xi$ and plot. Is the Jacobian positive everywhere?

4. Explain the best approximation property of the finite element method.