UNIVERSITY OF CALIFORNIA BERKELEY
Department of Civil Engineering
Spring 2009

Structural Engineering,
Mechanics and Materials
Professor: S. Govindjee

HW 2: Due Thursday Feb 12

1. Consider the following problem

$$\phi'' + \sin(\pi x) = 0$$

over the domain $x \in (0,1)$ with boundary conditions $\phi(0) = \phi(1) = 1$. Assume the following approximate trial function space

$$\mathcal{S}^h = \{\phi^h(x) \mid \phi^h(x) = 1 + \sum_{j=1}^{3} \phi_j \sin(\pi j x)\} \,.$$

(a) Show that the exact solution to this problem in an element of $\mathcal{S}^h$.

(b) Using the standard Bubnov-Galerkin prescription determine values for the B_js. Your test function space will be:

$$\mathcal{V}^h = \{v^h(x) \mid v^h(x) = \sum_{j=1}^{3} v_j \sin(\pi j x)\} \,.$$

Why? Note that, you don't need to do this by hand, it is also ok to do it numerically.

(c) Comment on the result you have obtained in part (b) relative to part (a).

2. Consider the functional

$$\Pi(\phi) = \int_0^1 [(\phi')^2 + x\phi]\, dx + 5\phi(1) \,.$$

Minimize this functional over the space of functions $\mathcal{S} = \{\phi \mid \phi(0) = 1\}$. Leave your result in the weak form.

3. Consider again problem 2. If you were to solve this problem with a finite element method with linear hat functions on a uniform mesh, approximately how many elements would be needed to ensure that the finite element error was less than 1% at all points in the domain? Hint: what is the absolute maximum value of $\phi''(x)$ in the domain?