HW 2: Due Thursday Feb 5

1. Formulate the weak form problem for the following strong form statement: Find $\phi(x)$ such that
$$\phi'' + 1 = 0 \qquad \forall x \in (0,1)\,,$$
where $\phi(0) = \phi(1) = 0$. Be sure to carefully define the space of trial solutions and the space of test functions.

2. Find an approximate (Bubnov-Galerkin) finite element solution to Problem 1. Assume finite element spaces composed of a single linear hat function centered at $x = 0.5$. Plot your solution against the exact solution.

3. Consider our canonical problem over a domain $(0, \pi)$. Assume that $AE = 1$, boundary conditions $u(0) = 0$ and $u'(\pi) = 1$, and a distributed load $b(x) = \delta(x - \pi/2)$. For a space of trial solutions assume:
$$\mathcal{S}^h = \{u^h(x) \mid u^h(x) = \sum_{A=1}^{N} u_A x^A\}$$
and assume a similar expression for $\mathcal{V}^h$.

   (a) Show that the linear equations governing the unknowns u_A have the form
   $$\sum_{A=1}^{N} K_{BA} u_A = F_B\,,$$
   where
   $$K_{BA} = \frac{AB}{A+B-1}\pi^{(A+B-1)}$$
   and
   $$F_B = \pi^B + (\pi/2)^B\,.$$

   (b) Using Matlab, solve these equations for $N \in \{1, 2, 3, 4, 5, 10, 50, 100, 200, 300, 400, 500\}$. (Note that you may not get solutions for the higher values of N depending upon your computer.)
      i. Plot your solutions against the exact solution.
      ii. Plot the error $e^h(x) = u_{\text{exact}}(x) - u^h(x)$ for each value of N.

iii. Plot the strain error $e^{h'}(x) = u'_{\text{exact}}(x) - u^{h'}(x)$ for each value of N.

iv. Plot the log of the L^2 norm of the error versus the log of N; i.e. make a plot of the L^2 norm of the error versus N on a log-log plot. When computing this norm, it is ok to use a discrete sum if you want instead of exactly computing the norm.

In this problem you will have a lot of plots. Organize the presentation of your results in a clear fashion. Do not forget to properly label the axes; use legends and titles on all plots or figure captions. Do not use a single page for each plot; you should probably put at least 4 to 6 plots on each page. You need to also make some comments on the behavior you see. Think and comment. Make a professional looking presentation of your results. Turning in a raw stack of plots will not get you any credit for this problem.