HW 1: Due Thurdays Jan 29

1. Consider the following ODE $\phi'' + x = 0$ over the interval $(0, 1)$ and the boundary conditions $\phi(0) = 0$ and $\phi'(1) = 1$. Find the exact solution and make a plot (to which you will add more curves in the next problems).

2. Consider the following guess for the solution to Problem 1 $\tilde{\phi} = Cx^2$, where C is a to-be-determined constant.

   (a) Find C such that the residual error at $x = 0.5$ is zero. Add this solution to your plot.

   (b) Find C such that the residual error at $x = 1.0$ is zero. Add this solution to your plot.

   (c) Find C such that the L^2 norm of the residual over $(0, 1)$ is minimized. Add this solution to your plot.

3. Compute the L^2 norm of the function $f(x) = x^2$ over the domain $\Omega = (0, 1)$.

4. Show that $\sin(x)$ and $\sin(2x)$ are orthogonal in the L^2 inner product over the domain $\Omega = (0, \pi)$

5. Consider the function $f_o(x) = 1$ over the interval $(0, 1)$. Find a linear function $f_1(x)$ with unit norm over the same interval such that f_1 is orthogonal to f_o; assume an L^2 norm and inner product.

Useful definitions:

1. L^2 inner product over the interval (a, b)

$$\langle f, g \rangle = \int_a^b f(x)g(x)\, dx$$

2. L^2 norm over the interval (a, b)

$$\|f\| = \sqrt{\langle f, f \rangle} = \sqrt{\int_a^b f^2\, dx}$$