

## Lab 6: 03/03/04

# Time Harmonic Heat Conduction

1. Time dependent heat conduction is governed by the PDE

$$\rho c \dot{T} + k \nabla^2 T = h_{tran}(T_{ext} - T) \quad . \quad (1)$$

We are interested in a harmonic solution of the form  $T(x, t) = \hat{T}(x)e^{i\omega t}$ . Plugging this into (1) gives

$$\rho c i \omega \hat{T} + k \nabla^2 \hat{T} = h_{tran}(\hat{T}_{ext} - \hat{T}) \quad . \quad (2)$$

Thus we can use the linear stationary heat equation to do a time harmonic analysis. Let the material be steel and consider having no convective terms in the domain. What should be our model parameters?

2. Consider the plate shown in figure 1 below. The curved boundary is subjected to the harmonic temperature  $T(x, t) = 100C e^{i\omega t}$ . Modell and solve the problem in femlab. How does the frequency  $\omega$  affect the solution?

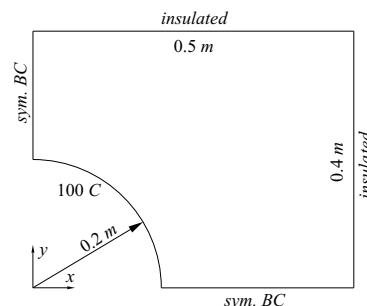


Figure 1: Heated Plate

3. Take  $\omega = 0.01$  and use the command `postsurf(fem, 'real(T)')` to visualize the real part of the solution. Also visualize `'imag(T)'`, `'abs(T)'` and `'angle(T)'`. Make proper annotations for your plots.
4. Plot  $\hat{T}(x)$  along the line through points (0.15,0.15) and (0.5,0.4) for the frequencies  $\omega = 0.0001, 0.001, 0.01, 0.1, 1$ .
5. Take the point  $\mathbf{x} = (0.3, 0.3)$  and plot  $\log |T(\mathbf{x})|$  versus  $\log \omega$ . Can you interpret the curve?
6. Do you have an idea what we are doing here?