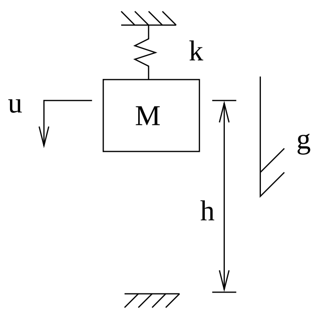
UNIVERSITY OF CALIFORNIA BERKELEY
Department of Civil Engineering
Spring 2004

Structural Engineering,
Mechanics and Materials
Professor: S. Govindjee

HW 1: Due 2/4/04

1. Spring-Mass:

$$\Pi(u) = \frac{1}{2}ku^2 + Mg(h-u) \tag{1}$$



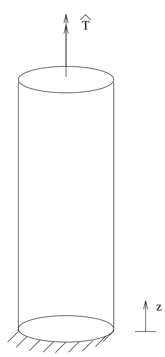
   (a) Consider the spring-mass system shown. Plot the potential energy of the system versus the spring displacement and verify the potential is a minimum at static equilibrium.

   (b) Take the derivative of Π, apply the necessary condition for an extremal value, and derive the static equlibrium equations.

2. Consider a functionally graded torsion bar as shown; the potential energy is given by

$$\Pi(\phi(z)) = \int_0^L \frac{1}{2}G(z)J(d\phi/dz)^2\,dz - \hat{T}\phi(L)\,, \tag{2}$$

   where J is the polar moment of inertia, $G(z) = G_1 + G_2 z$ is a spatially varying shear modulus, and ϕ is the section rotation.



   Assume

$$\mathcal{S} = \left\{ \phi \mid \phi = \left\{ \begin{array}{ll} Az & z \leq L/2 \\ B(z-L/2) + AL/2 & z > L/2 \end{array} \right. \right\}. \tag{3}$$

Find $\phi \in \mathcal{S}$ which minimizes Π. Compare to the exact solution $\phi(z) = \frac{\hat{T}}{JG_2} \ln(1 + \frac{G_2}{G_1} z)$.

3. Consider the following potential for an elastic bar which is loaded by a force P at $x = L$, a body force $b(x)$, and held fixed at $x = 0$.

$$\Pi(u(x)) = \int_0^L \frac{1}{2} AE(du/dx)^2 \, dx - \int_0^L bu \, dx - Pu(L) \,. \tag{4}$$

Take the variational derivative of Π and derive the classical governing equation:

$$AE\frac{d^2u}{dx^2} + b = 0 \tag{5}$$