ETH Zurich
Department of Mechanical and Process Engineering
Winter 06/07
Nonlinear Continuum Mechanics
Exercise 11

Institute for Mechanical Systems
Center of Mechanics

Prof. Dr. Sanjay Govindjee

1 Homework

1.1 Burgers fluid

Derive the corresponding ordinary differential equation governing the behaviour of the Burgers fluid. A schematic of the model is shown in Figure 1.

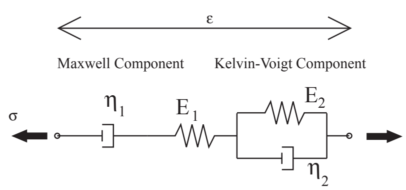

Figure 1: Schematic of Burgers Fluid

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1.2 Storage and loss moduli

Consider a 1-D GSLS that is subject to a strain history of

$$\varepsilon(t) \quad = \quad \varepsilon_0 \sin(\omega t) \; . \tag{1}$$

The steady state response can be written as,

$$\sigma(t) \quad = \quad \varepsilon_0 \left[G'(\omega) \sin(\omega t) + G''(\omega) \cos(\omega t) \right] \; , \tag{2}$$

where $G'(\omega)$ is the storage modulus and $G''(\omega)$ is the loss modulus. Find the expressions for the storage and loss moduli for a GSLS with two Maxwell elements. A schematic of this model is shown in Figure 2.

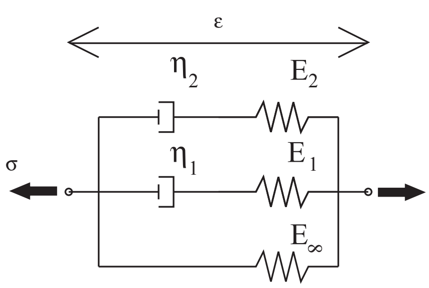

Figure 2: Schematic of GSLS with two Maxwell elements

Assume $E_\infty = E_1 = E_2 = 1$, and $\eta_1 = 1$ and $\eta_2 = 100$. Plot $G'(\omega)$,$G''(\omega)$ versus $\log_{10}(\omega)$ over the range $[-3, 3]$. Interpret the distinctive features of the curves.

1.3 Viscoelastic Poisson ratio

In viscoelastic materials the Poisson function is defined to be,

$$\nu(t) \quad = \quad -\frac{\varepsilon_{22}(t)}{\varepsilon_{11}(t)} \tag{3}$$

in a uniaxial constant load experiment where a cylinder of material is exposed to a step stress in the 1-direction and $\nu(t)$ is the ratio of the axial strain to the transverse strain. Assume a material that has elastic bulk behavior and Maxwell fluid deviatoric behavior:

$$
\begin{aligned}
p &= K\text{tr}[\varepsilon] && (4)\\
p &= \frac{1}{3}\text{tr}[\sigma] && (5)\\
\dot{e}_{ij} &= = \frac{\dot{s}_{ij}}{2\mu} + \frac{s_{ij}}{\eta} && (6)\\
s_{ij} &= \sigma_{ij} - p\delta_{ij} && (7)\\
e_{ij} &= \varepsilon_{ij} - \frac{1}{3}\text{tr}[\varepsilon] && (8)
\end{aligned}
$$

and find the Poisson function. Evaluate your answer for the two limits $\nu(0)$ and $\nu(\infty)$. Note: the *given* stress state is simply,

$$
\begin{aligned}
\boldsymbol{\sigma}(t) &\rightarrow H(t)\begin{bmatrix} \sigma_0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} && (9)\\
H(t) &= \begin{cases} 1 & (t \geq 0) \\ 0 & (t < 0) \end{cases} && (10)
\end{aligned}
$$

and you need to determine the strain response. Insert this stress into the given partial differential equations and solve the corresponding ODE. You may but you DO NOT have to use the linear operators that I introduced in the lecture.