Argument for $\theta_{A+B} = \theta_B = \theta_B$: A Temperature

Consider two systems A and B that are independently in equilibrium with their own heat bathes such that they have canonical distributions

$$\rho_A(\boldsymbol{y}_A) = C_A \exp[-H_A/\theta_A] \tag{1}$$
$$\rho_B(\boldsymbol{y}_B) = C_B \exp[-H_B/\theta_B] \,. \tag{2}$$

Assume now that the two systems are brought together and now have a distribution

$$\rho_{A+B}(\boldsymbol{y}_A, \boldsymbol{y}_B) = C_{A+B} \exp[-H_{A+B}/\theta_{A+B}] \,. \tag{3}$$

Suppose during this process, A and B remain in equilibrium – nothing changes. Then

$$\rho_{A+B} = \rho_A \rho_B \,. \tag{4}$$

Let us now assume weak interaction and integrate both sides of eqn (4) over the degrees of freedom of A we get

$$\frac{C_{A+B}}{\tilde{C}_A} \exp[-H_B/\theta_{A+B}] = C_B \exp[-H_B/\theta_B] \,, \tag{5}$$

where $\tilde{C}_A = \int_{\Gamma_A} \exp[-H_A/\theta_{A+B}]$. Recall that weak interaction says that $H_{A+B} = H_A + H_B$ for the purposes of integrations over phase space. Since eqn (5) has to hold for all values of $\boldsymbol{y}_B$ one find that $\theta_{A+B} = \theta_B$. Similarly if one integrates both sides of eqn (4) over Γ_B, one finds $\theta_{A+B} = \theta_A$. Thus θ functions exactly like a (absolute) temperature.