UNIVERSITY OF CALIFORNIA BERKELEY   Structural Engineering,
Department of Civil Engineering   Mechanics and Materials
Spring 2012   Professor: S. Govindjee

1 Two properties of the Dirac Delta Function

The Dirac delta function is a well known generalized function (or distribution) such that

$$\int \delta(x - x_o) f(x) dx = f(x_o) \tag{1}$$

under suitable conditions on the function $f(\cdot)$.

   Property 1: If one scales the argument of the Dirac delta function then the result is simply scaled; i.e.

$$\int \delta(a(x - x_o)) f(x) dx = \frac{1}{a} f(x_o) \,. \tag{2}$$

To prove this result simply consider a change of variables $y = ax$ and apply eqn (1).

   Property 2: Consider a function $f(x)$ with zeros[1] at x_i such that $f(x) \approx f'(x_i)(x - x_i)$ for x near x_i, then

$$\int \delta(f(x)) g(x) dx = \sum_i \frac{1}{f'(x_i)} g(x_i) \,. \tag{3}$$

The proof follows directly from eqn (2).

---

[1]Such zeros are sometimes called simple zeros.