UNIVERSITY OF CALIFORNIA BERKELEY
Department of Civil Engineering
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Structural Engineering,
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Handy Facts for Problems in Statistical Mechanics

Useful integrals:

$$\int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \sqrt{\frac{\pi}{\alpha}}$$

$$\int_{0}^{\infty} e^{-\alpha x^2} dx = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}}$$

$$I(n) = \int_{0}^{\infty} e^{-\alpha x^2} x^n dx = \frac{1}{2}\Gamma\left(\frac{n+1}{2}\right)\alpha^{-(n+1)/2} = -\frac{\partial I(n-2)}{\partial \alpha}$$

The gamma-function:

$$\begin{aligned}
\Gamma(z) &= \int_0^\infty e^{-x} x^{z-1} dx \\
\Gamma(z+1) &= z\Gamma(z) \\
\Gamma(n) &= (n-1)\Gamma(n-1) = (n-1)! \\
\Gamma(1) &= 1 \\
\Gamma(1/2) &= \sqrt{\pi} \\
\Gamma(1-z)\Gamma(z) &= \frac{\pi}{\sin(\pi z)} \\
\Gamma(z)\Gamma(z+\frac{1}{2}) &= 2^{1-2z}\sqrt{pi}\Gamma(2z)
\end{aligned}$$

The error-function:

$$\mathrm{erf}(y) = \frac{2}{\sqrt{\pi}} \int_0^y e^{-x^2} dx$$

Alternate form for the factorial

$$n! = \int_0^\infty e^{-x} x^n dx = \Gamma(n+1)$$

Stirling's formula:

$$n! = \sqrt{2\pi n}\; n^n e^{-n} \left[1 + \frac{1}{12n} + \cdots\right] \qquad \text{for } n \gg 1\,.$$

Thus for $n > 10$ the error in using only the pre-factor is already only 1 percent. For very large n , $\ln(n) \ll n$, this reduces to $\ln(n!) \approx n\ln(n) - n$.

Useful expression for the Kronecker-Delta:

$$\delta_{nm} = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{i(n-m)\theta} d\theta$$

Useful expressions for the Dirac-Delta function:

$$\delta(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-ikx} dk\,.$$

This expression simply comes from the inverse Fourier transform expression for the delta function.

Geometric Series

$$\sum_{i=0}^{n} af^i = a\frac{1-f^{n+1}}{1-f}$$