UNIVERSITY OF CALIFORNIA BERKELEY
Department of Civil Engineering
Fall 2013

Structural Engineering,
Mechanics and Materials
Professor: S. Govindjee

Midterm Exam: CE235 / ME279
Due Friday 10/22/2013

Problem	Score
#1	/20
#2	/40
#3	/40
Total	/100

____
Name

____
SID

Permitted Materials: Class notes, books, and necessary computing environments (e.g. Matlab, Mathematica, etc.).

Honor Code: I have not given or received aid in this examination. I have taken an **active part** in seeing to it that others as well as myself uphold the spirit and letter of this Honor Code.

____
(Signature)

1. Consider the (bSpace) notes on the Gibbs paradox and additivity of the thermodynamic functions. Provide a discussion with quantitative estimates for the box-gas to ensure the additivity assumption, say for entropy or other thermodynamic functions, holds.

2. In computing the properties of crystals, we have made use of the Born – von Karmen periodicity assumption. Consider a one-dimensional crystal, with a single atom in the basis, and discuss the impact of the periodic boundary condition versus the use of alternate boundary conditions, for example fixed-fixed or free-free boundary conditions. Provide quantitative estimates to back up your statements. Be sure to discuss what happens as $N \to \infty$.

3. Consider a system that consists of two elements in contact with a heat bath of temperature T. Each element has a single degree of freedom μ_i ($i = \{1, 2\}$)that takes on the value of either $+1$ of -1 (spin-up or spin-down). The elements of the system are massless and thus have no kinetic contribution to the Hamiltonian. Further assume that the system is subject to an external (controlled) field B that interacts with the states of the system such that the Hamiltonian of the system can be expressed as:

$$H = -(\mu_1 + \mu_2)B + \alpha\mu_1\mu_2 \,,$$

where $\alpha > 0$ and $B > 0$ are given.

(a) Enumerate the four possible micro-states for the system.

(b) Show that the partition function can be expressed as $Z(B, \alpha, T) = 2e^{-\alpha/kT}\cosh(2B/kT) + 2e^{\alpha/kT}$.

(c) Determine the phase average $\overline{\mu_1\mu_2}$ as a function of non-dimensional temperature ($\theta = kT/2B$) and the ratio α/B.

i. What happens in the limit of large θ?

ii. What happens in the limit of $\theta \to 0$? List all the possible cases.

iii. Make a plot of $\overline{\mu_1\mu_2}$ versus θ and describe what is happening in the system as it cools.