UNIVERSITY OF CALIFORNIA BERKELEY
Department of Civil Engineering
Fall 2013

Structural Engineering,
Mechanics and Materials
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HW 1: Due September 12

Some thinking exercises

1. Consider a fair coin. What is the probability that one will obtain the sequence $HHTT$? How many possible 4 flip sequences are there?

2. Suppose a coin is not fair with a probability of a head of p and a tail of q, with $p + q = 1$. If the coin is flipped N times, what is the probability of getting $N/4$ or more heads? [Hint: Binomial distribution].

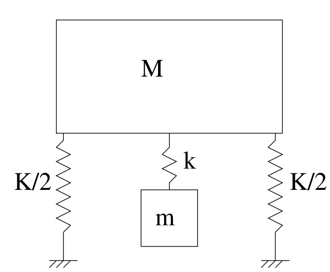
3. The probability that a welder will make a mistake on a single 30 cm long strip-weld can be modeled using a Poisson distribution (equivalent to the binomial distribution in the rare event case). In this case, one can write $p(n) = \frac{\lambda^n}{n!}\exp[-\lambda]$ for the probability the welder makes n errors in the strip-weld, where λ is a model parameter. Assuming that $\lambda = 0.02$, what is the probability that a given strip-weld has no errors? What is the probability that a given strip-weld contains at least 3 errors.

4. (Bayes' Theorem) Bayes' theorem tells us that the conditional probability of an event B given an event A is
$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}\,.$$
I sit on a train next to a person with long hair. What is the probability that that person was a woman. Note that the population is split evenly between men and women. Further 75% of women have long hair and only 5% of men have long hair.

   [Hint: Apply Bayes' theorem where event B is the event women and event A is the event that someone has long hair. You should also exploit the fact that $P(A) = \sum_i^N P(A|B_i)P(B_i)$ where the union $\cup_{i=1}^N B_i = \Omega$, where Ω is the set of all possible events, and $B_i \cap B_j = \emptyset$ for $i \neq j$]. It also possible to solve this problem by constructing a probability event tree and then making a direct counting of possibilities.

Hamiltonian mechanics exercises

5. Consider the mechanical system shown. Each mass moves in the vertical direction only. What is the Hamiltonian for the system? What are the (1st order) equations of motion for the system?



6. Consider a Hamiltonian for a system that can be described by two generalized coordinates q_1 and q_1. Related to these coordinates are two generalized momenta p_1 and p_2. Thus we have $H(q_1, q_2, p_1, p_2, t)$. Suppose that we have the additional result that $\partial H/\partial q_2 = 0$. What quantity is conserved for all motions of this system?

Statistical mechanics exercises

7. Given a phase space distribution $\rho(\boldsymbol{q}, \boldsymbol{p}, t) : \Gamma \to \mathbb{R}$ of the form $\rho(\boldsymbol{q}, \boldsymbol{p}, t) = \hat{\rho}(H(\boldsymbol{q}(t), \boldsymbol{p}(t)))$, show that this represents a distribution which is *always* in statistical equilibrium; i.e., show $\partial\rho/\partial t = 0$.

8. Consider a dynamical system described by the Hamiltonian

$$H(q,p,t) = \frac{1}{2m}p^2 + \frac{k_o[1+\sin^2(\pi t)]}{2}q^2$$

and the phase function $F(q,p) = p^2$. Find dF/dt.