UNIVERSITY OF CALIFORNIA BERKELEY
Department of Civil Engineering
Fall 2010

Structural Engineering,
Mechanics and Materials
Professor: S. Govindjee

Midterm Exam: CE235 / ME279
Due Friday 10/8/2010
11:10am

Problem	Score
#1	/20
#2	/40
#3	/40
Total	/100

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Name

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SID

Permitted Materials: Class notes, books, and necessary computing environments (e.g. Matlab, Mathematica, etc.).

Honor Code: I have not given or received aid in this examination. I have taken an **active part** in seeing to it that others as well as myself uphold the spirit and letter of this Honor Code.

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(Signature)

1. Consider the Helium atom in a gravitational field from Homework 3. Compute

   (a) The coefficient of variation ($\sqrt{\overline{\Delta z^2}}/\overline{z}$) for its height.

   (b) The coefficient of variation of the energy.

2. Consider N non-interacting particles with mass m inside a spherical vessel with radius a. The particles are subject to a potential energy $U_o \ln(r/a)$, where $U_o > 0$ is a given constant and r is the distance from the origin (center of the vessel). The vessel is in thermal equilibrium with a heat bath at temperature T.

   (a) Write an expression for the system Hamiltonian that takes takes into account the fact that the particles are confined to sphere of radius a.

   (b) Determine the partition function for the system (canonical ensemble). [Hint: Express momenta and coordinates in spherical coordinates to make the computations easier. It is ok to assume the system sufficiently warm so that the integrals exist.]

   (c) What is the Helmholtz free energy for the system?

   (d) What is the entropy of the system?

   (e) Find the macroscopic pressure-volume relation for the system.

   (f) Find the heat capacity for the gas.

   (g) Considering the mean radial position of a particle in gas, what occurs as one cools the system towards $T \searrow U_o/3k$.

3. Consider a system that consists of two elements in contact with a heat bath of temperature T. Each element has a single degree of freedom μ that takes on the value of either $+1$ of -1 (spin-up or spin-down). The elements of the system are massless and thus have no kinetic contribution to the Hamiltonian. Further assume that the system is subject to an external (controlled) field B that interacts with the states of the system such that the Hamiltonian of the system can be expressed as:

$$H = -(\mu_1 + \mu_2)B + \alpha\mu_1\mu_2 \,,$$

   where $\alpha > 0$ and $B > 0$ are given.

   (a) Enumerate the four possible micro-states for the system.

   (b) Show that the partition function can be expressed as $Z(B, \alpha, T) = 2e^{-\alpha/kT}\cosh(2B/kT) + 2e^{\alpha/kT}$.

   (c) Determine the phase average $\overline{\mu_1\mu_2}$ as a function of non-dimensional temperature ($\theta = kT/2B$) and the ratio α/B.

      i. What happens in the limit of large θ?

      ii. What happens in the limit of $\theta \to 0$? List all the possible cases.

      iii. Make a plot of $\overline{\mu_1\mu_2}$ versus θ and describe what is happening in the system as it cools.