UNIVERSITY OF CALIFORNIA BERKELEY
Department of Civil Engineering
Fall 2010

Structural Engineering,
Mechanics and Materials
Professor: S. Govindjee

HW 7: Optional

1. Consider a spherical particle moving with *uniform* speed in a fluid with viscosity η. The frictional retarding force f must then be a function of its radius a, velocity v and η (it can not depend on the density of the fluid since the motion is uniform). Use dimensional analysis to find the functional form for f (modulo a constant of proportionality) and show it is consistent with Stoke's solution.

2. Consider the Langevin equation

$$\frac{dv}{dt} = -\frac{\alpha}{m}v + \frac{1}{m}R(t)\,.$$

   (a) Show that the solution to this equation can be written as:

$$v(t) = v_o e^{-\alpha t/m} + \frac{1}{m}e^{-\alpha t/m}\int_0^t e^{\alpha s/m}R(s)\,ds\,,$$

   where $v_o = v(0)$ – the initial condition.

   (b) Show that the correlation function $\langle v(0)v(t)\rangle = \langle v^2(0)\rangle e^{-\alpha|t|/m}$, where the angle brackets denote ensemble averaging at fixed times t.[1]

   (c) Write $R(t)$ and $v(t)$ as Fourier integrals and show how their Fourier coefficients (Fourier transforms) must be related (by the Langevin equation); i.e. find the relation between the 'spectral densities' of $R(t)$ and $v(t)$.

   (d) Find the spectral density of $v(t)$ using the correlation function from Part (2b).

   (e) Combine these results to determine a relation for the spectral density of $R(t)$.

3. Show that for the Langevin model that

$$\alpha = \frac{1}{2kT}\int_{-\infty}^{\infty}\langle R(0)R(s)\rangle\,ds\,.$$

   This is one form of the celebrated fluctuation-dissipation theorem.

[1] This result is often expressed by additionally using the equipartition theorem to replace the mean square velocity by kT/m.