HW 1: Due September 10

Some thinking exercises

1. Consider a fair coin. What is the probability that one will obtain the sequence HHH? How many possible 3 flip sequences are there?

2. Suppose a coin is not fair with a probability of a head of p and a tail of q, with $p + q = 1$. If the coin is flipped N times, what is the probability of getting $N/4$ or more heads? [Hint: Binomial distribution].

3. The probability that a typesetter will make a mistake on a single page in a book can be modeled using a Poisson distribution (equivalent to the binomial distribution in the rare event case). In this case, one can write $p(n) = \frac{\lambda^n}{n!}\exp[-\lambda]$ for the probability the typesetter makes n errors on a single page, where λ is a model parameter. Assuming that $\lambda = 1$, what is the probability that a given page has no errors? What is the probability that a given page contains at least 3 errors.

4. (Bayes' Theorem) Bayes' theorem tells us that the conditional probability of an event B given an event A is
$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}\,.$$
Suppose AMD manufactures computer chips in two plants. Plant 1 makes 60% of its chips and Plant 2 40% of its chips. If 25% of the chips from Plant 1 are defective and 10% of the chips from Plant 2 are defective, what is the probability that a randomly selected defective chip came from Plant 1? [Hint: Apply Bayes' theorem where event B is the event Plant 1 and event A is the event that you have a defective chip. You should also exploit the fact that $P(A) = \sum_i^N P(A|B_i)P(B_i)$ where the union $\cup_{i=1}^N B_i = \Omega$, where Ω is the set of all possible events, and $B_i \cap B_j = \emptyset$ for $i \neq j$]. It also possible to solve this problem by constructing a probability event tree and then making a direct counting of possibilities.

Statistical mechanics exercises

5. Consider two masses, m_1, m_2, that are connected by a linear spring with spring constant k and reference length l that slide on a frictionless table. Assuming the motion only occurs in one-dimension, construct the Hamiltonian for this system, $H(x_1, x_2, p_1, p_2)$, and Hamilton's form of the equations of motion.

6. Given a phase space distribution $\rho(\boldsymbol{q}, \boldsymbol{p}, t) : \Gamma \rightarrow \mathbb{R}$ of the form $\rho = \hat{\rho}(H(\boldsymbol{q}, \boldsymbol{p}, t))$, show that this represents a distribution which is *always* in statistical equilibrium; i.e., show $\partial\rho/\partial t = 0$.