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Steady State Response of the SLS

Let $\varepsilon(t) = \varepsilon_o \sin(\omega t)$, then for the standard linear solid (SLS)

$$\sigma(t) = \int_{-\infty}^{t} [E_\infty + (E_o - E_\infty)\exp(-(t-s)/\tau)]\varepsilon_o \omega \cos(\omega s)\, ds \quad (1)$$

$$= \varepsilon_o E_\infty \sin(\omega t) + \varepsilon_o \omega (E_o - E_\infty)\exp(-t/\tau)\int_{-\infty}^{t} \exp(s/\tau)\cos(\omega s)\, ds \quad (2)$$

$$= \varepsilon_o E_\infty \sin(\omega t) + \varepsilon_o \omega (E_o - E_\infty)\exp(-t/\tau)\left[\frac{\exp(s/\tau)}{1/\tau^2 + \omega^2}\left[\frac{1}{\tau}\cos(\omega s) + \omega \sin(\omega s)\right]\right]_{-\infty}^{t} \quad (3)$$

$$= \varepsilon_o E_\infty \sin(\omega t) + \frac{\varepsilon_o \omega (E_o - E_\infty)}{1/\tau^2 + \omega^2}\left[\frac{1}{\tau}\cos(\omega t) + \omega \sin(\omega t)\right] \quad (4)$$

$$= \varepsilon_o \underbrace{\left[E_\infty + \frac{\omega^2\tau^2}{1+\omega^2\tau^2}(E_o - E_\infty)\right]}_{E'(\omega)} \sin(\omega t) + \varepsilon_o \underbrace{(E_o - E_\infty)\frac{\omega\tau}{1+\omega^2\tau^2}}_{E''(\omega)} \cos(\omega t) \quad (5)$$

The storage modulus, $E'(\omega)$, gives the amount of "stiffness in phase with the applied loading" and $E''(\omega)$ gives the amount of "stiffness 90 degrees out of phase with the loading". Note the in the limit of $\omega >> \tau$, $E'' \to 0$ and $E' \to E_o$. Whereas in the limit $\omega << \tau$, $E'' \to 0$ and $E' \to E_\infty$. Out of phase, i.e. viscous behavior, is seen for frequencies comparable to $1/\tau$. One common measure of viscous dissipation is the so-called loss angle, δ, defined through

$$\tan(\delta) = E''/E' \quad (6)$$