

HW #2: CE231 / MSE211

As we have seen we can express vectors in terms of their components with respect to a particular basis. Thus we can write

$$\mathbf{v} = v_i \mathbf{e}_i. \quad (1)$$

When the particular basis is understood (ie. everyone reading the equation will assume the same basis) one can safely express the components of the vector in a “column vector”; ie. as

$$v_i \rightarrow \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}. \quad (2)$$

This convention also applies to second order tensors. In this case we have for a basis $e = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$

$$\mathbf{T} = T_{ij} \mathbf{e}_i \otimes \mathbf{e}_j. \quad (3)$$

If the basis is understood we can express the components in a 3×3 matrix. The following ordering is conventional:

$$T_{ij} \rightarrow \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{bmatrix} \quad (4)$$

For the problems assume an orthonormal basis $e = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ and

$$\mathbf{a} = 2\mathbf{e}_1 + 5\mathbf{e}_2 - 7\mathbf{e}_3, \quad (5)$$

$$\mathbf{b} = 0\mathbf{e}_1 - 8\mathbf{e}_2 + 1\mathbf{e}_3, \quad (6)$$

$$c_i \rightarrow \begin{pmatrix} 4 \\ 5 \\ 7 \end{pmatrix}, \quad (7)$$

and

$$T_{ij} \rightarrow \begin{bmatrix} 1 & 8 & 2 \\ 8 & 3 & 2 \\ 2 & 2 & 3 \end{bmatrix}. \quad (8)$$

Find the following. Express your answers in dyadic and matrix/column vector form (assume the e basis). When the answer is a scalar, just give the number.

1. $(\mathbf{a} \cdot \mathbf{b})\mathbf{c}$
2. $b_3 \mathbf{a} \cdot \mathbf{c}$

3. $(\mathbf{a} \cdot \mathbf{b})\mathbf{a} \otimes \mathbf{b}$
4. δ_{ii}
5. $T_{3j}\delta_{3j}$
6. $T_{ij}\delta_{ij}$
7. $T_{ij}T_{ij}$
8. \mathbf{Tc}
9. I_T
10. II_T
11. III_T
12. $\mathbf{e}_1 \otimes \mathbf{e}_2$
13. $\mathbf{e}_3 \otimes \mathbf{e}_2$