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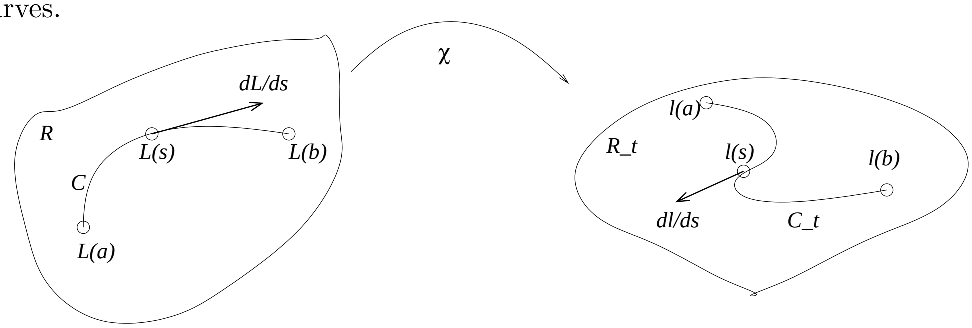
Mappings of tangents to curves

In lecture we showed that the deformation gradient, F_{iA}, maps 'local' vectors from the reference configuration to the current (or spatial) configuration. The result is approximate in the sense that $F_{iA} dX_A = dx_i + \text{h.o.t.}$[1]. In all of our constructions we always focus on the properties at a point and thus implicitly always consider the limit as $dX_A \rightarrow 0$. Sometimes, however, is is useful to have an expression for the mapping of certain vectors independent of their magnitudes. Such vectors are the tangent vectors to material curves.



Consider a reference configuration curve $\mathcal{C}$ defined by the points $\boldsymbol{L} : [a,b] \rightarrow \mathcal{R}$, where $[a,b]$ is an arbitrary interval of the real line, $\mathbb{R}$. At each point $s \in [a,b]$ of the curve, the tangent vector to the curve is given by $d\boldsymbol{L}(s)/ds$. After deformation, every point of $\mathcal{C}$ is mapped to $\mathcal{C}_t$ by the deformation map, thus defining a new curve $\boldsymbol{l} : [a,b] \rightarrow \mathcal{R}_t$, where $\boldsymbol{l}(s) = \boldsymbol{\chi}(\boldsymbol{L}(s))$. The tangent vector at each point of this new curve is given by:

$$\frac{d\boldsymbol{l}}{ds} = \frac{\partial \boldsymbol{\chi}}{\partial \boldsymbol{X}} \cdot \frac{d\boldsymbol{L}}{ds} \qquad \frac{dl_i}{ds} = \frac{\partial \chi_i}{\partial X_A}\frac{dL_A}{ds} \quad (1)$$

$$= \boldsymbol{F} \cdot \frac{d\boldsymbol{L}}{ds} . \qquad = F_{iA}\frac{dL_A}{ds} . \quad (2)$$

From this last result, we can observe that *without approximation of any form!* the tangent vector to a curve of material points in the reference configuration is mapped by the deformation gradient to a tangent vector of a curve in the spatial configuration composed of the same material points. For

[1] h.o.t. means higher order terms and in this context terms that are quadratic or higher in the norm of dX_A.

this reason, in some books you will find the deformatino gradient referred to as the tangent map.