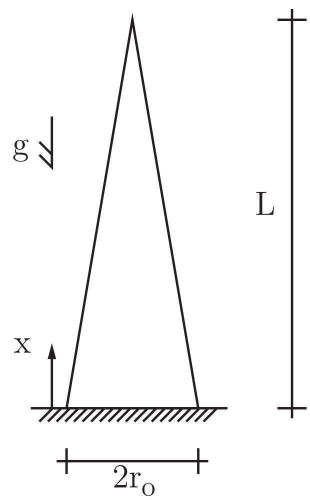
**HW 9 Due Wednesday April 9**

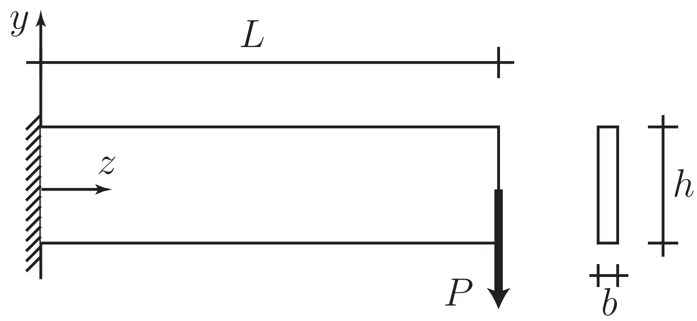
[Problems numbers are from revision (e) of the book.]

1. (10 pts) Book Problem 5.13

2. (30 pts) Consider an Aluminum, vertical, tapered circular column of height $L$, radius $r(x) = r_o - r_o x/L$, density $\rho$, clamped at the base and free at the top. Determine a restriction on the height of the column. For properties, let $E = 11$ GPa, $r_o = 0.3$ m, $\rho = 2700$ kg/m$^3$, and $g = 9.81$ m/s$^2$. Assume $\tilde{\mathcal{S}} = \{v(x) \mid v(x) = c_1 x^2 + c_2 x^3\,, \quad c_1, c_2 \in \mathbb{R}\}$. [Hint: Compute the necessary integrals numerically. The determinant equation will result in a high degree polynomial; solve for its roots numerically and then select the physically meaningful root.]




3. (30 pts) Consider a deep cantilever beam $h \gg b$ of length $L$. If such a beam is loaded with a tip-load $P$, it will bend (very slightly) in the $y, z$-plane – bending about the $x$-axis. However, if the load $P$ exceeds a critical value $P_{cr}$, the beam will undergo a so-called lateral-torsional instability – a form of buckling in which the beam twists about the $z$-axis while it simultaneously bends about the $y$-axis.




The potential energy for the system (with respect to twisting) is given by the relation:

$$\Pi[\phi(z)] = \int_0^L \frac{1}{2}(GJ)_{\text{eff}}(\phi'(z))^2\,dz - \int_0^L \frac{P^2}{2EI_y}(L-z)^2\phi^2(z)\,dz\,, \tag{1}$$

where $(GJ)_{\text{eff}} = \frac{1}{3}hb^3G$ is the torsional rigidity and $I_y = hb^3/12$ is the area moment of inertia about the $y$-axis. Assume $\tilde{\mathcal{S}} = \{\phi(z) \mid \phi(z) = C(z^2 - 2Lz), \quad C \in \mathbb{R}\}$ and find an approximation for $P_{cr}$.

[Remark: The exact buckling mode is given by $\phi(z) = \sqrt{L-z}J_{-1/4}\left(2.0063(L-z)^2/L^2\right)$ with $P_{cr} = \frac{4.013}{L^2}\sqrt{EI_y(GJ)_{\text{eff}}}$ – see e.g. §6.3 in S.P. Timoschenko and J.M Gere, *Theory of Elastic Stability*, McGraw-Hill (1961). The function $J_{-1/4}$ is the Bessel function of the first kind of order $-1/4$. It can be evaluated/plotted in MATLAB using the built-in function `besselj( )`.]