UNIVERSITY OF CALIFORNIA BERKELEY
Department of Civil Engineering
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Structural Engineering,
Mechanics and Materials
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HW 11: Due April 21

1. (15pt) Consider a circular membrane of radius R which is restrained from deflection at its perimeter. The membrane is under a uniform tension of magnitude S and loaded with a uniform pressure p. Assume a deflection of the form:

$$w(r,\theta) = C\left[1 - \left(\frac{r}{R}\right)^2\right]$$

and using stationary potential energy find an approximate expression for the center-point deflection. Note that in polar coordinates

$$\nabla w = \begin{pmatrix} \frac{\partial w}{\partial r} \\ \frac{1}{r}\frac{\partial w}{\partial \theta} \end{pmatrix}$$

and that $\int_A (\cdot) dx dy \rightarrow \int_A (\cdot) r dr d\theta$.

2. (15pt) Consider a rectangular plate $\Omega = \{(x,y) \mid 0 \leq x \leq a \text{ and } 0 \leq y \leq b\}$. The plate is loaded at its center with a point force P (transverse to the plane of the plate). In terms of its deflection, $w(x,y)$ (z-direction), the strain energy of the plate can be expressed as:

$$\Pi_{\text{elastic}} = \int_\Omega \frac{D}{2}\left[\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2 + 2\nu\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial y^2}\right) + 2(1-\nu)\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right] dA\,,$$

where $D = Eh^3/[12(1-\nu^2)]$ is the flexural rigidity of a plate with thickness h. Assume that the plate is supported at its edges with simple supports (no displacement, free to rotate) and compute an approximate solution for the deflection of the plate of the form:

$$w(x,y) \approx C\sin(\pi x/a)\sin(\pi y/b)$$

using stationary potential energy.

3. (15pt) Consider a plate of dimension $2a \times 2b$ with clamped edges (zero motion and zero rotation) which is loaded with a uniform transverse pressure p_o. Assume an approximate solution in the form of a cosine function in x times a cosine function in y (coordinate origin at the center of the plate) and find an approximate expression for the center point displacement. [Hint: The exact solution $w(0,0) \approx 0.0759 p_o a^4/Eh^3$ for $a/b = 1.5$ and $\nu = 0.3$. Your solution should be quite close to this.]