Mechanics of Structures (CE130N)
Lab 9

1 Objective

The objective of this lab is to apply the principle of stationary potential energy for the solution of beam buckling problems.

2 Buckling load: A cantilever beam (clamped-free case)

Figure 1: Cantilever beam

Consider a cantilever beam subjected to an axial compressive load. The potential energy for this system is

$$\Pi(v(x)) = \int_0^L \frac{1}{2} EI(v'')^2 \, dx - P \int_0^L \frac{1}{2}(v')^2 \, dx \,. \tag{1}$$

If we assume the deflection has the polynomial form

$$v(x) = \sum_{i=1}^{N} c_i \left(\frac{x}{L}\right)^{i+1} \,, \tag{2}$$

then the kinematic boundary conditions at $x = 0$ are automatically satisfied.

2.1 Exercise 1

If we insert this approximation into the expression for the potential energy, then we can write the governing equilibrium equations as $(\boldsymbol{K} - \lambda \boldsymbol{G})\boldsymbol{c} = \boldsymbol{0}$, where $\boldsymbol{K}$ comes from the bending energy term, $\boldsymbol{G}$ comes from the potential for the load, and $\boldsymbol{c}$ is the vector of c_is divided by the length. Here $\lambda = PL^2/EI$ is a non-dimensional load value. Show that

1. $$K_{ij} = \frac{(i+1)(j+1)ij}{i+j-1} \tag{3}$$

2. $$G_{ij} = \frac{(i+1)(j+1)}{i+j+1} \tag{4}$$

2.2 Exercise 2

Download the file `lab9_1_student.m` from the bspace. This file partially implements a solution to this buckling problem. Where indicated complete the file as follows:

1. Fill in the appropriate expressions for $\boldsymbol{K}$ and $\boldsymbol{G}$.
2. Extract the critical non-dimensional buckling load λ_{cr} and store in `approx`.
3. Extract the corresponding eigenvector and store in `c`.

2.3 Exercise 3

1. What is the error with just one term in the approximation?
2. What is the error with 3 terms in the approximation?
3. Make a plot of log error versus number of parameters in approximation for $n = 1, 2, \ldots, 10$.

3 Beam with a transverse load and axial compression

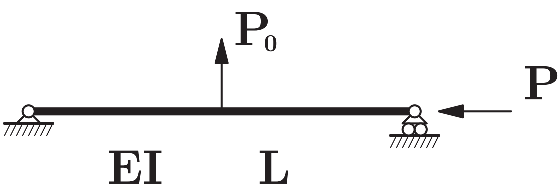

Figure 2: Simply supported beam with transverse load and axial compression.

Consider the beam shown in the figure. It is subjected to an axial compression P and a transverse load P_o. The potential energy for this system is given by

$$\Pi(v(x)) = \int_0^L \frac{1}{2} EI(v'')^2\, dx - P \int_0^L \frac{1}{2}(v')^2\, dx \; - P_o v(x_o)\,, \tag{5}$$

where x_o is the location where the transverse load is applied. If we assume the deflection has the trigonometric form

$$v(x) = \sum_{i=1}^{N} c_i \sin(i\pi x/L)\,, \tag{6}$$

then the resulting discrete equations have the form $(\boldsymbol{K} - P\boldsymbol{G})\boldsymbol{c} = \boldsymbol{F}$, where the stiffness $\boldsymbol{K}$ and geometric stiffness $\boldsymbol{G}$ matrices have a very simple form (they are diagonal).

3.1 Exercise 4

1. Determine expressions for K_{ij} and G_{ij}.

2. Determine an expression for F_i.

3. Since your matrices are diagonal one can find an expression for c_i by hand.

3.2 Exercise 5

Download the file `lab9_2_student.m` from bspace. This file partially computes a solution this problem. Complete the file as follows:

1. Where indicated implement your expression for c_i.

2. Where indicated complete the expression that evaluates the buckling mode for plotting purposes.

3.3 Exercise 6

1. Find and plot the deflection of the beam for a transverse load $P_o = 3$ and axial load $P = 0.2P_{\text{Euler}}$, where $x_o = 3L/5$.

2. Plot the midspan deflection of the beam (with $P_o = 3$) versus $P \in [0,\, 0.95P_{\text{Euler}}]$. When you make your plot, use $v(L/2)$ for the abscissa and P the ordinate.

3. Add to this plot curves for $P_o = 1$ and $P_o = 5$

Provide a detailed description of what the plots tell you.