UNIVERSITY OF CALIFORNIA BERKELEY
Department of Civil Engineering
Spring 2010

Structural Engineering,
Mechanics and Materials
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Mechanics of Structures (CE130N)
Lab 8

1   Objective

The objective of this lab is to understand how the principle of stationary potential energy can be applied to understand the behavior of buckling phenomenon. In this lab we will focus the case of a discrete system consisting of rigid elements and springs.

2   Geometry

The geometry we will work with is the two degree of freedom shown below. The two degrees of freedom that we will work with are the rotation angles of the bars from vertical. The bars are joined with a linear torsional spring with spring constant k_t which has dimensions of Force × Length. Further the middle joint is restrained by a linear translational spring with spring constant k_l which has dimensions of Force / Length.
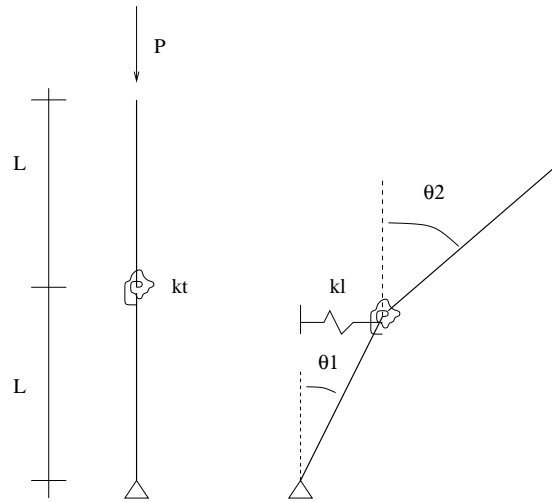
3   Potential Energy

The potential energy for this system can be expressed as

$$\Pi(\theta_1, \theta_2) = \frac{1}{2}k_l L^2[(1 - \cos(\theta_1))^2 + \sin^2(\theta_1)] + \frac{1}{2}k_t(\theta_2 - \theta_1)^2 - PL(2 - \cos(\theta_1) - \cos(\theta_2)). \quad (1)$$

This expression is valid independent of the magnitude of the rotations.

3.1 Non-dimensionalization

In many problems, it is useful to non-dimensionalize expressions to remove redundant parameters from the problem and to improve performance in numerical calculations. The dimensions of energy are Forces × Length thus a convenient parameter for non-dimensionalization is k_t. This yields

$$\Pi(\theta_1, \theta_2) = \frac{1}{2}e[(1 - \cos(\theta_1))^2 + \sin^2(\theta_1)] + \frac{1}{2}(\theta_2 - \theta_1)^2 - \lambda(2 - \cos(\theta_1) - \cos(\theta_2)), \quad (2)$$

where the potential energy is now non-dimensionalized (divided by k_t), $e = k_l L^2 / k_t$ is a non-dimensional ratio of spring stiffnesses, and $\lambda = PL/k_t$ is a non-dimensional load parameter.

3.2 Exercise 1

Approximate the energy to quadratic order in the rotations. Keeping up to quadratic order terms in the potential energy leads to linear order equilibrium equations. Hint:

$$\cos(x) \approx 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \cdots \quad (3)$$

$$\sin(x) \approx x - \frac{1}{3!}x^3 + \cdots \quad (4)$$

3.3 Exercise 2

Compute the equilibrium equations associated with your approximate energy and arrange them in the matrix form $\boldsymbol{K} - \lambda \boldsymbol{G} = \boldsymbol{0}$.

3.4 Exercise 3

Download the Matlab files from bspace and place them in your working directory. These files include plotting routines as well as a file which computes the potential energy of the system for large deformations.

Compute (using Matlab) the buckling loads and modes using [V,D]=eig(K,G). To help you visualize the results you should use the routine plotv.m. Typing help plotv explains how to use the routine. For example to plot the first buckling mode in red type:

```
>> plotv(V,D,1,L,'r')
```

where L is the length of the bars (choose 10 for example) and the vectors and eigenvalues are stored in V and D.

1. Assuming $e = 0.1$, what is the critical non-dimensional buckling load? and corresponding bucking mode?

2. Assuming $e = 5.0$, what is the critical non-dimensional buckling load? and corresponding bucking mode?

3. Assuming $e = 100.0$, what is the critical non-dimensional buckling load? and corresponding bucking mode?

4   Energy Landscape

For every value of the load parameter, the potential energy possesses stationary points that represent the possible equilibrium states of the system. An equilibrium point can either be a minimum point (i.e. stable), a saddle point (i.e. unstable in one direction, stable in another), or a maximum point (i.e. unstable). Consider the case of $e = 10$. Make plots for the following three load cases using `plotpe.m`. Type `help plotpe` to see the arguments for this routine. For each case, identify all the equilibrium points on the graph and label them as stable or unstable.

4.1   Exercise 4

1. $\lambda = 0.80\lambda_{\text{cr}}$.

2. $\lambda = 5.00\lambda_{\text{cr}}$.

3. $\lambda = 30.0\lambda_{\text{cr}}$.

Note, `plotpe.m` plots the energy (2) and not the approximate energy.

5   Evolution of equilibrium

To be able to visualize the development of the system's equilibrium states one can animate the progression of the equilibrium states with increasing loads. To do so, use the routine `animate8.m`. The routine takes two arguments `e` the stiffness ratio and `L` the bar lengths. Before being able to use the routine, you will need to edit the routine to add expressions for $\boldsymbol{K}$ and $\boldsymbol{G}$ where indicated.

To use type:

```
>> animate8(e,L)
```

where `e` and `L` are set to appropriate values.

The routine will plot the buckling modes was well as a surface plot of the energy and a contour plot of the energy. The contour plot will additionally have the directions of the linearized buckling modes superposed – red being the first mode and blue the second mode. The animation plots the state of energy of the system for ten load values below the critical load, then ten load values between the two buckling loads, and finally ten values above the second buckling load.

5.1 Exercise 5

Animate the system for the case $e = 5$ (and $L = 10$).

1. By observing the plot, describe the expected evolution of the system as the load is increased from zero to the highest value.

2. Are there ever any stable equilibrium states associated with the second buckling mode?