UNIVERSITY OF CALIFORNIA AT BERKELEY
Department of Civil and Environmental Engineering
Structural Engineering, Mechanics and Materials

CE 130N, Spring 2009
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Lab 11

Mechanics of Structures (CE130N)
Lab 11

1 Objective

The objective of this lab is to understand how the principle of stationary potential energy can be applied to understand the behavior of buckling phenomenon in beams.

2 Exercise

2.1 Download files

1. Download the file `ritz_buckling.zip` into your `ce130n/programs` directory and unzip it.
2. Go to the `ce130n/programs/ritz_buckling/exercise/` directory, and execute the file `init.m`. This will set the necessary paths to run the files.

YOU MUST RUN THE FILE init.m EVERYTIME YOU START UP MATLAB.

2.2 Buckling load: A simply supported beam

Run the example which computes the buckling load of a simply supported beam shown in Figure 1. The data structure for this mechanical structure is defined in, `buckling_ex2_2.m`. To run this example one can type the following lines of MATLAB code.

```
>> buckling_ex2_2;                         % -- Load data structure
>> plotproblem_buckling(ritz,buckling);    % -- Plot problem
>> plotritzf(ritz);                        % -- Plot Ritz functions
>> K = computeK(ritz);                     % -- Compute stiffness matrix
>> B = computeB(ritz,buckling);            % -- Compute buckling matrix
>> [V,D] = eig(K,B);                       % -- Compute the generalized
>>                                         %    eigenvalues
>> c = V(:,1);                             % -- Specify c as the first eigenvector
>> plotsol(ritz,c);                        % -- Plot buckling mode shape
>> D(1,1)                                  % -- Display the eigenvalue
```

The functions used previously in the Ritz program are all valid for the buckling problem, i.e., `plotsol.m`, `plotritzf.m`, `evaldisp.m` and others.

This problem uses the sine function,

$$\begin{aligned} v(x) &= cf(x), \\ f(x) &= \sin\left(\pi \frac{x}{L}\right) , \end{aligned}$$

to compute the approximate critical buckling load (eigenvalue) and buckling mode shape (eigenvector). How good is this approximation? Why is it good or bad?

In the data structure file, one can change the number `ritz.N` to increase the number of functions used in the approximation. Does the critical load change as you increase the number of functions used in the approximation? Why or why not? How many terms are required to obtain a good approximation?

Figure 1: Simply supported beam

2.3 Buckling load: A cantilever beam

Modify the file `buckling_ex2_2.m` to compute the buckling load of a cantilever beam shown in Figure 2. Use polynomials to find the approximate critical load. How many terms are required to obtain a critical load which has a relative error of 1×10^{-8}.(Look at your notes for the exact solution.)

Figure 2: Cantilever beam

2.4 Buckling load: A fixed-pinned beam

Modify the file `buckling_ex2_2.m` to compute the buckling load of a fixed-pinned beam shown in Figure 3. Think of a good function $f(x)$ for the deflection,

$$v(x) \quad = \quad cf(x),$$

which satisfies the kinematic B.C. to compute the approximate critical buckling load (eigenvalue) and buckling mode shape (eigenvector). How good is your approximation in terms of the relative error? The exact solution to the buckling load is,

$$P_{cr} \quad = \quad (4.493409457909065)^2 \frac{EI}{L^2} \, .$$

Figure 3: Fixed-pinned beam

2.5 Load deflection curve: A simply supported beam

Consider the simply-supported beam with a vertical load P_0 at the center and an axial load P shown in Figure 4. The data structure for this problem is defined in `buckling_ex2_5.m`. A Ritz approximation using sine functions is employed,

$$
\begin{aligned}
v(x) &= \sum_{i=1}^{N} c_i f_i(x), \\
f(x) &= \sin\left(\pi \frac{x}{L}\right) .
\end{aligned}
$$

One can determine the coefficients $\mathbf{c}$ by solving the system of equations,

$$(\mathbf{K} - P\mathbf{B})\mathbf{c} = \mathbf{F} \ ,$$

where $\mathbf{K}$ is the stiffness matrix, $\mathbf{B}$ is the buckling matrix, and $\mathbf{F}$ is the forcing vector. Note that for this case $\mathbf{F}$ is non-zero. The coefficients $\mathbf{c}$ can be determined through MATLAB with the code,

```
>> buckling_ex2_5;                        % -- Load data structure
>> plotproblem_buckling(ritz,buckling);  % -- Plot problem
>> plotritzf(ritz);                       % -- Plot Ritz functions
>> K = computeK(ritz);                    % -- Compute stiffness matrix
>> B = computeB(ritz,buckling);           % -- Compute buckling matrix
>> F = computeF(ritz);                    % -- Compute buckling matrix
>> P = 1;                                 % -- Assume an axial load of 1
>> c = (K-P*B)\F;                         % -- Compute coefficients
>> plotsol(ritz,c);                       % -- Plot buckling mode shape
```

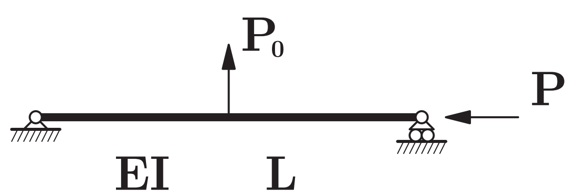

Figure 4: Simply supported beam with load

As one increases the load P from zero, the deflection of the beam will also increase. Plot the relationship between the axial load P and displacement at the middle of the beam $v(L/2)$ for $P_0 = 1$. The key characters of the plot should be identifed. Make sure you have enough terms in your approximation for a sufficiently accurate answer. What happens to this plot when P_0 is increased or decreased?

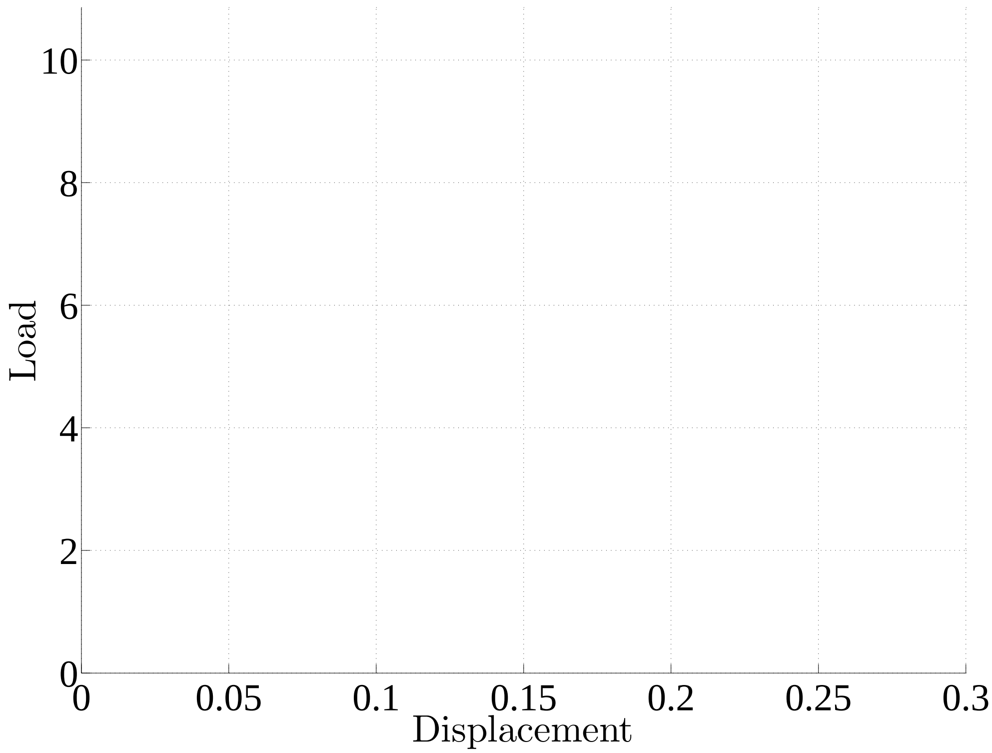

Figure 5: Load displacement curve

2.6 Buckling load: A simply supported beam with spring

Consider the simply-supported beam with a spring in the middle shown in Figure 6. The data structure for this problem is defined in `buckling_ex2_6.m`. A Ritz approximation using sine functions is employed,

$$
\begin{aligned}
v(x) &= \sum_{i=1}^{N} c_i f_i(x), \\
f(x) &= \sin\left(\pi \frac{x}{L}\right) .
\end{aligned}
$$

As you will see through your homework, the addition of the spring results in a change in the stiffness matrix $\mathbf{K}$. Define $\mathbf{K}_{\text{beam}}$ as the stiffness matrix in the presence of only the beam, and define $\mathbf{K}_{\text{spring}}$ as the additional contribution arising from the spring. The stiffness matrix for the joint beam and spring structure becomes,

$$
\mathbf{K} = \mathbf{K}_{\text{beam}} + \mathbf{K}_{\text{spring}} .
$$

Compute the critical load for the case of $k_t = 1$ and $k_t = 200$. Draw the buckling mode shapes for each case.

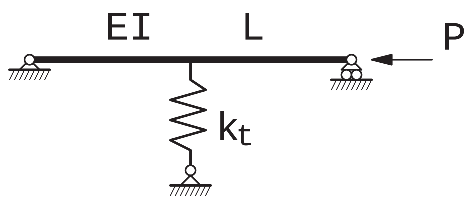

Figure 6: Simply supported beam with spring

2.7 EXTRA: Buckling load for distributed loads

Consider the cantilever beam with a uniform distributed axial load solved in class. The data structure for this problem is defined in `buckling_extra.m`. A Ritz approximation using polynomials is employed,

$$\begin{aligned} v(x) &= \sum_{i=1}^{N} c_i f_i(x), \\ f_i(x) &= \left(\frac{x}{L}\right)^{i+1} . \end{aligned}$$

Observe how the critical load converges to the exact load.