UNIVERSITY OF CALIFORNIA BERKELEY
Department of Civil Engineering
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Structural Engineering,
Mechanics and Materials
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**HW 11: Due Thurday May 7**

1. Consider a square membrane with side length $a$ which is clamped at its perimeter and subject to a uniform tension $S$ and a transverse pressure $p(x,y)$.
   (a) Derive the weak form expression for this problem.
   (b) Find a solution for the deflection of membrane by approximately solving the weak form. Use single parameter solution and test spaces.

2. Consider a (Bernoulli-Euler) beam of length $L$ and bending stiffness $EI$ subject to a transverse loading $q(x)$ and boundary conditions $\theta(0) = \theta_o$, $V(0) = V_o$, $V(L) = V_L$, and $\theta(L) = \theta_L$.
   (a) Derive (from the strong form) the weak equilibrium equation for the beam.
   (b) Select a test function that allows you to determine from the weak equilibrium equation a condition on $q(x)$ for the system to be in a state of equilibrium. You should be able to separately argue that this condition makes sense.

3. Consider the torsional response $T(\phi') = T_c \mathrm{sign}(\phi') \, |\phi'/\phi'_c|^n$, where $T_c = 100$ Nm, $\phi'_c = 0.01$ rad/m, and $n = 2$.
   (a) Plot the torque versus twist rate from 0 to $5\phi'_c$.
   (b) Consider a bar of length $L = 0.75$ m which is built-in at $z = 0$ and subjected to a distributed torque $t(z) = t_c = 150$ Nm/m.
      i. Show that the exact rotation field of the bar is

$$\phi(z) = \frac{\phi'_c}{(t_c/T_c)(1+\frac{1}{n})} \left( \frac{t_c L}{T_c} \right)^{1+\frac{1}{n}} \left[ 1 - \left( \frac{L-z}{L} \right)^{1+\frac{1}{n}} \right] \tag{1}$$

      and plot it.
      ii. Use an approximation for the solution space and the test function space to find an approximate solution for this problem from the weak form. Include a sufficient number of parameters to arrive at a reasonable solution.
      iii. Now assume that the bar is built-in at both ends and compute an approximate solution to the weak form.