

**HW 10: Due Thursday April 23**

1. Derive, starting from  $dT/dz + t = 0$ , the weak equilibrium equation for a bar of length  $L$  in torsion that is subjected to a constant distributed torque  $t(z) = t_o$ , an applied end-rotation  $\phi(0) = \bar{\phi}$ , an applied torque  $T_L$  at  $z = L$ .
2. Consider a tension-compression bar with  $N$  applied forces  $F_i$  at  $x = iL/N$  for  $i = 1, 2, \dots, N$ . What is the virtual work equation? Assume the bar is built in at  $x = 0$ .
3. Consider the case in Problem 2, assume that  $N = 3$  and that  $F_i = i\bar{F}$  for some constant  $\bar{F}$ . Find an approximate solution to the weak form by using a sub-space of functions that includes linear and quadratic terms. Compare your result to the exact answer.
4. Consider the potential energy for the system described in Problem 1. Compute the variational derivative of the energy. Your answer should match the result you got in Problem 1.
5. Consider the potential energy for a stretched membrane of domain  $\Omega$  with tension  $S$  and applied transverse pressure  $p(x, y)$ . Assume the membrane is fixed on its boundary  $\partial\Omega$ . Compute the variational derivative of the energy.