Finite Element Solution of Contact Problems: From 1974 to 2004

by

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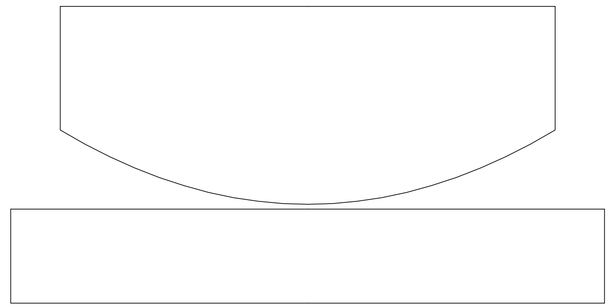
Remarks on FEM Contact Analysis

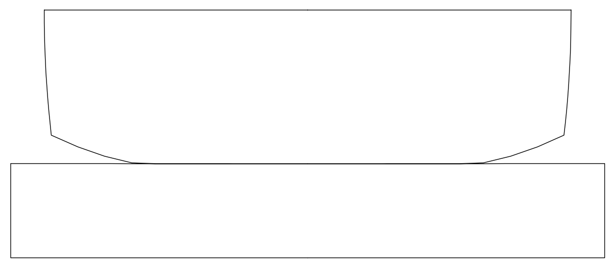
- This talk summarizes:
  - Formulations to treat **contact** problems by FEM.
  - Spatial approximation for **contact** and **tied** interfaces.
  - Methods of solution using **Lagrange multiplier**, **perturbed lagrangian**, **penalty**, **augmented lagrangian** and **constraint elimination** methods.
  - Contact **patch test** requirement.
  - Example solutions for various treatments.

Finite Element Contact Analysis

- Multiple body problems can have two states:



(a) No contact condition.



(b) Contact state.

1974-77: The Foundations: With TJRH!

THE BODIES ARE DISCRETIZED USING STANDARD FINITE ELEMENT TECHNIQUES

CONTACT SURFACE :

COORDINATES IN DEFORMED BODIES

$$\int_c \boldsymbol{\tau} \cdot (\mathbf{x}^1 - \mathbf{x}^2)\, dc$$

LAGRANGE MULTIPLIER (= CONTACT TRACTION)

$$\int_c \{ \delta\boldsymbol{\tau} \cdot (\mathbf{x}^1 - \mathbf{x}^2) + \boldsymbol{\tau} \cdot \delta\mathbf{x}^1 - \boldsymbol{\tau} \cdot \delta\mathbf{x}^2 \}\, dc$$

ASSUME $\boldsymbol{\tau}$ CONSISTS OF A SET OF N DISCRETE VECTORS :

$$\sum_{i=1}^{3} \sum_{j=1}^{N} \tau_{ij} (x_{ij}^1 - x_{ij}^2)$$

i — SPATIAL DIRECTION

τ_{ij} — NODAL CONTACT FORCE

1974-77: The Foundations: With TJRH!

EXAMPLE : FRICTIONLESS CASE

$$\int_c \tau \,(x^1 - x^2)\, dc$$

τ = NORMAL COMPONENT OF CONTACT FORCE

x^α = NORMAL COMPONENT OF DEFORMED SURFACE COORDINATE IN BODY α

DISCRETE VERSION:

$$\sum_{j=1}^{N} \tau_j \,(x_j^1 - x_j^2)$$

τ_j = NODAL CONTACT FORCE

x_j^α = NODAL COORDINATE

AT A TYPICAL NODE j

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} \quad \text{CONTACT 'STIFFNESS'}$$

MULTIDEGREE - OF - FREEDOM SYSTEMS :

Lumped — USE NEWMARK METHOD

$$\begin{bmatrix} \mathbf{M}^1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{M}^2 \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{x}}^1 \\ \mathbf{0} \\ \ddot{\mathbf{x}}^2 \end{bmatrix} + \begin{bmatrix} \mathbf{K}^1 & \bigcirc & \mathbf{0} \\ \bigcirc & \square & \bigcirc \\ \mathbf{0} & \bigcirc & \mathbf{K}^2 \end{bmatrix} \begin{bmatrix} \mathbf{x}^1 - \mathbf{x}^1 \\ \boldsymbol{\tau} \\ \mathbf{x}^2 - \mathbf{x}^2 \end{bmatrix} = \begin{bmatrix} \mathbf{F}^1 \\ \bigcirc \\ \mathbf{F}^2 \end{bmatrix}$$

Σ FORM AND PREFACTOR

WHERE :

$\mathbf{M}^\alpha$ = MASS MATRIX BODY α

$\mathbf{K}^\alpha$ = STIFFNESS MATRIX BODY α

$\mathbf{F}^\alpha$ = APPLIED FORCE VECTOR BODY α

$\mathbf{0}$ = ZERO MATRIX OR VECTOR

$\boldsymbol{\tau}$ = VECTOR OF CONTACT FORCES

ETC.

NODES WHICH ARE IN CONTACT CONTRIBUTE TO $\bigcirc$

NODES WHICH ARE NOT IN CONTACT CONTRIBUTE TO $\square$

THE EQUATIONS ARE NONLINEAR SECOND - ORDER ORDINARY DIFFERENTIAL EQUATIONS

1974-77: The Foundations: With TJRH!

AT IMPACT AND RELEASE SPECIAL CONSIDERATION MUST BE GIVEN TO VELOCITY AND ACCELERATION :

EXAMPLE :

ASSUME A PAIR OF NODES IMPACT AT STEP $n+1$.

NEWMARK FORMULAS :

$$\underset{\sim}{a}^{\alpha}_{n+1} = \frac{1}{\beta}\left\{ \frac{1}{\Delta t^2}\left(\underset{\sim}{d}^{\alpha}_{n+1} - \underset{\sim}{d}^{\alpha}_{n} - \Delta t\,\underset{\sim}{v}^{\alpha}_{n}\right) - \left(\tfrac{1}{2}-\beta\right)\underset{\sim}{a}^{\alpha}_{n} \right\}$$

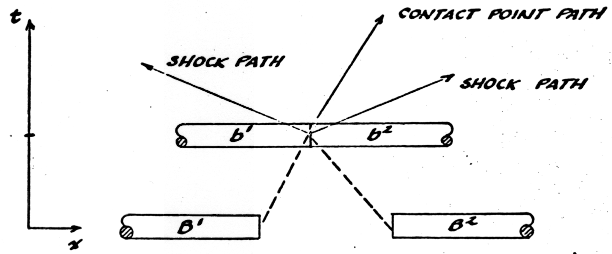
$$\underset{\sim}{v}^{\alpha}_{n+1} = \underset{\sim}{v}^{\alpha}_{n} + \Delta t\left\{ (1-\gamma)\underset{\sim}{a}^{\alpha}_{n} + \gamma\,\underset{\sim}{a}^{\alpha}_{n+1} \right\}$$

$\Rightarrow$ (IN GENERAL) $\underset{\sim}{a}^{1}_{n+1} \neq \underset{\sim}{a}^{2}_{n+1}$

$\underset{\sim}{v}^{1}_{n+1} \neq \underset{\sim}{v}^{2}_{n+1}$

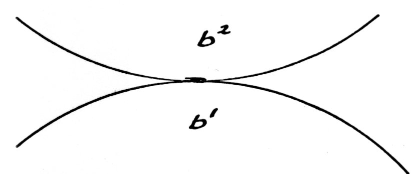
THIS VIOLATES THE PHYSICS !

1974-77: The Foundations: With TJRH!



CAN SOLVE FOR THE POST-IMPACT STATE EXPLICITLY

THREE - DIMENSIONAL CASE :



APPROXIMATE LOCALLY BY HALF - SPACE SOLUTION

CAN OBTAIN POST - IMPACT VELOCITY AND STRESS VIA THIS PROCEDURE

↑ NOT A USEFUL QUANTITY

1974-77: The Foundations: With TJRH!

MORAL : POST - IMPACT PROBLEM IS ONE OF WAVE PROPAGATION AND IS ESSENTIALLY ONE - DIMENSIONAL

THE VELOCITY FROM THE LOCAL WAVE PROPAGATION ANALYSIS :

$$v_+ = \frac{\rho_0^2 U^2 v_{-1}^2 - \rho_0^1 U^1 v_{-1}^1}{\rho_0^2 U^2 - \rho_0^1 U^1}$$

ρ_0^α = DENSITY IN REFERENCE CONFIGURATION

U^α = MATERIAL VELOCITY OF SHOCK WAVE

(E.G., IN LINEAR ELASTICITY U^α IS 'BAR - WAVE' VELOCITY IN 1-DIM. AND DILATATIONAL VEL. IN 3-DIM.)

THE ERRONEOUS ACCELERATIONS SATISFY

$$M^\alpha a_-^\alpha + K^\alpha(d_-^\alpha) - (-1)^\alpha \tau_- = 0$$

WHERE

$$a_-^1 \neq a_-^2$$

WE WANT

$$M^\alpha a_+ + K^\alpha(d_-^\alpha) - (-1)^\alpha \tau_+ = 0$$

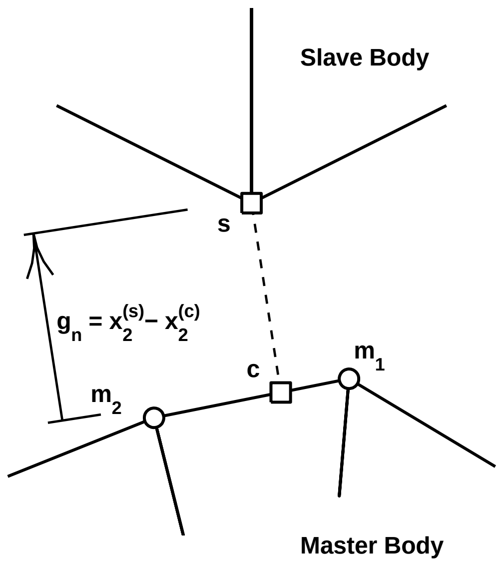
SOLVING :

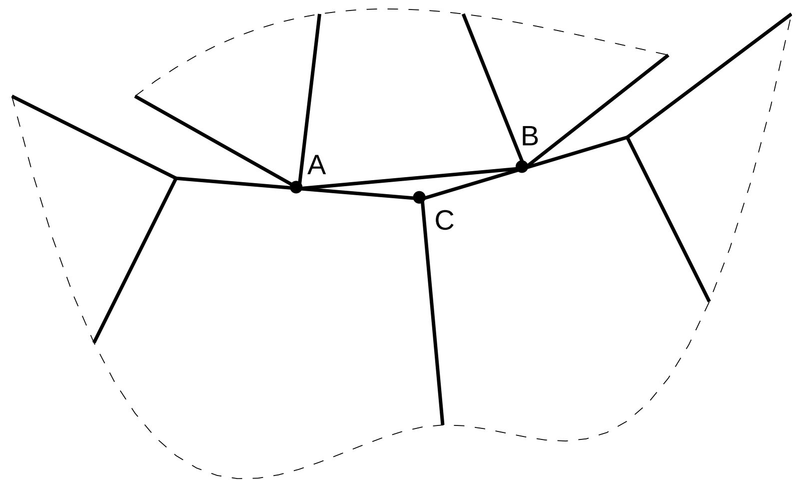
$$a_+ = \frac{M^1 a_-^1 + M^2 a_-^2}{M^1 + M^2}$$

$$\tau_+ = \tau_- - \frac{M^1 M^2}{(M^1 + M^2)}(a_-^2 - a_-^1)$$

1982-89: Node-to-Surface Contact

- Simplest form where nodes on one body do not interact at nodes on second body (Goudreau & Hallquist, 1982)

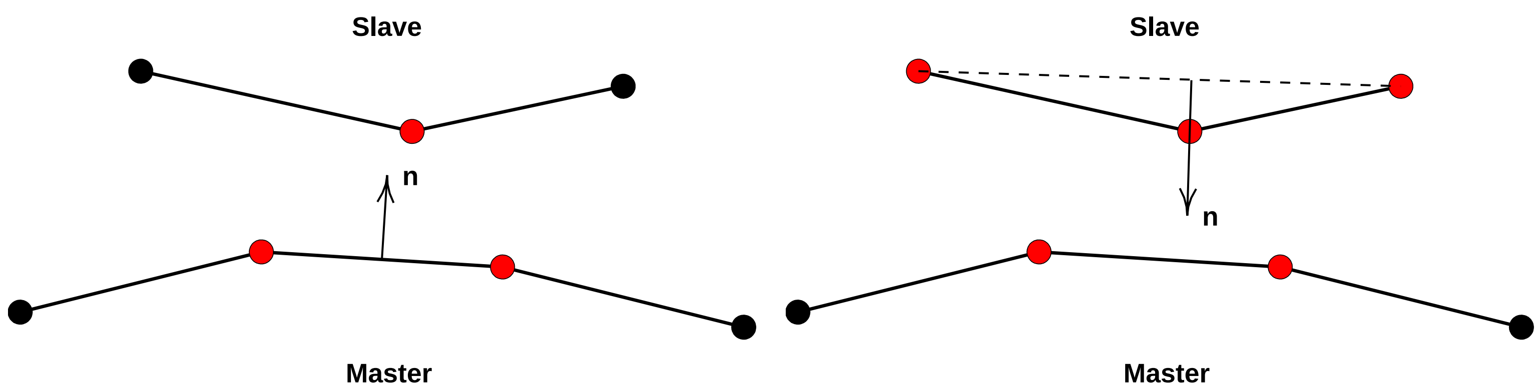




- Note nodes of master body <u>not</u> in contact.
- Consistent tangent (Simo & Wriggers, 1985; Parische, 1989)

1982-89: Node-to-Surface Contact

- Normals from FE mesh:



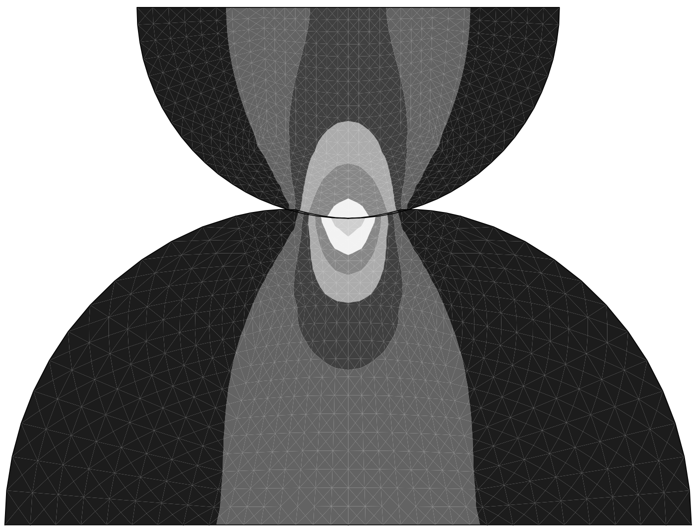
(a) Normal to Master. (b) Normal to Slave.

- Note master form involves 3 nodes & slave form 5 nodes.

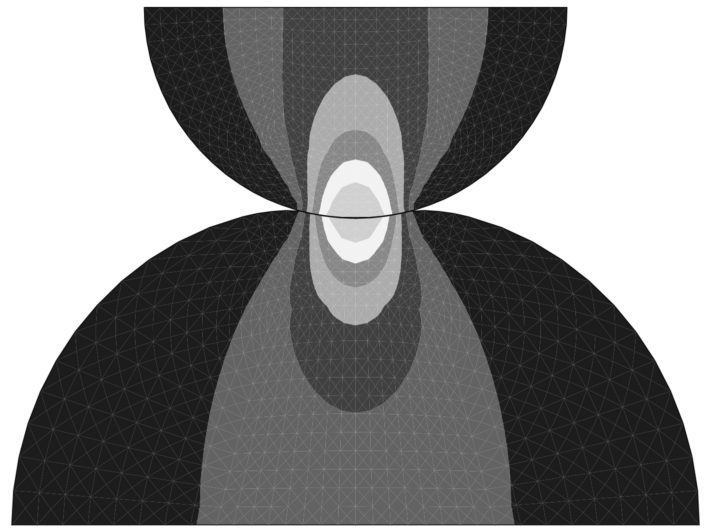
- Slave normal gives **smoother** sliding.

1982-1989: Node-to-Surface Contact

- Example: Compare σ_{22} for treatments



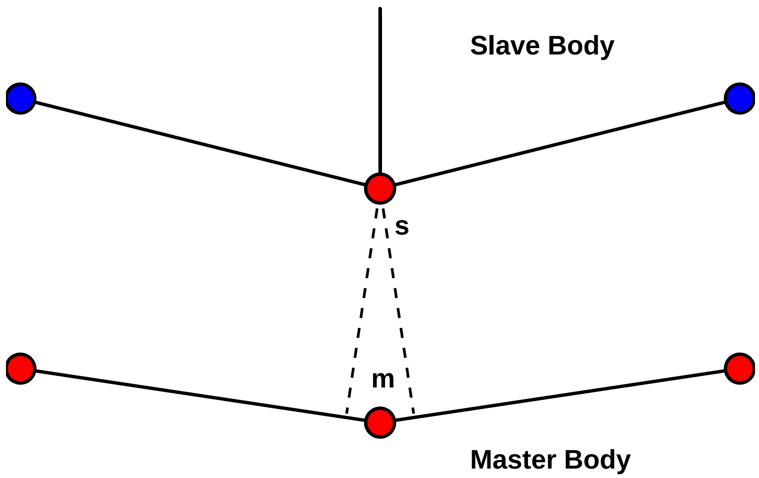
(a) Node to node



(b) Node to surface

1982-1989: Node-to-Surface Contact

- Difficulty: Corner Condition

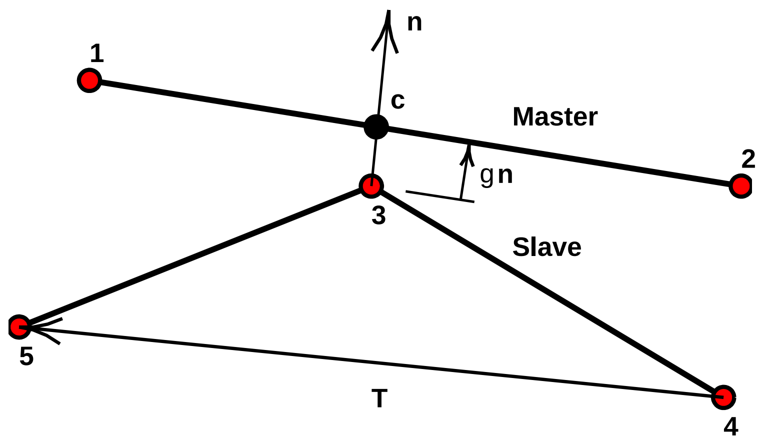


- Use two facets and modify functional to

$$\Pi_c = \sum_{i=1}^{2} \left\{ (\lambda_i + \lambda_{ki}) \mathbf{n}_{ci}^T \left[\mathbf{x}_s - \mathbf{x}(\xi_i) \right] - \frac{1}{\kappa} \lambda_i^2 \right\}$$

1982-1989: Node-to-Surface Contact

- Difficulty: Slave normal not perpendicular to master facet.



- For smooth sliding must ignore $\delta\xi$ term and let

$$\delta\Pi_c = \left[\ \delta\lambda_n,\ \ \delta\mathbf{x}_s^T,\ \ \delta\mathbf{x}_\alpha^T\ \right] \left\{ \begin{array}{c} g_n\, A_c \\ \lambda_n\, A_c\, \mathbf{n}_c \\ -\,\lambda_n\, A_c\, N_\alpha\, \mathbf{n}_c \end{array} \right\}$$

  Gives slave force on 3-node only.

- Tangent **unsymmetric**.

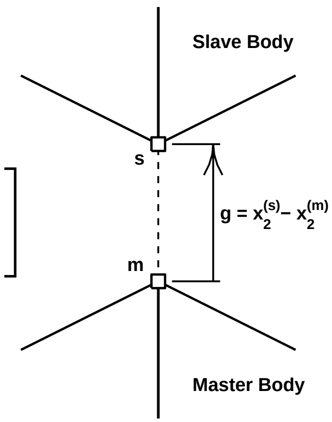
1985: Solution Methods

- Methods for imposing contact constraint (Landers & T, 1985):
  - **Lagrange multiplier** method.
  - **Perturbed lagrangian**, **penalty** and **perturbed tangent** methods (related).
  - **Augmented lagrangian** method of Uzawa.
  - **Constraint elimination** method.

1985: Solution Methods

- **Lagrange multiplier** form of solution

$$\Pi_c = \lambda\, g = \lambda \left[(X_2^{(s)} + u_2^{(s)}) - (X_2^{(m)} + u_2^{(m)}) \right]$$



- Added variational equation

$$\delta\Pi_c = \delta\lambda\, g + \left(\delta u_2^{(s)} - \delta u_2^{(m)} \right) \lambda = \left[\ \delta u_2^{(s)} \quad \delta u_2^{(m)} \quad \delta\lambda\ \right] \left\{ \begin{array}{r} \lambda \\ -\lambda \\ g \end{array} \right\}$$

- λ - **contact force** to prevent penetration.

1985: Solution Methods

- Linearization of Π_c produces tangent matrix

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix} \begin{Bmatrix} du_2^{(s)} \\ du_2^{(m)} \\ d\lambda \end{Bmatrix} = \begin{Bmatrix} -\lambda \\ \lambda \\ -g \end{Bmatrix}$$

- Added identical to any FE assembly process.

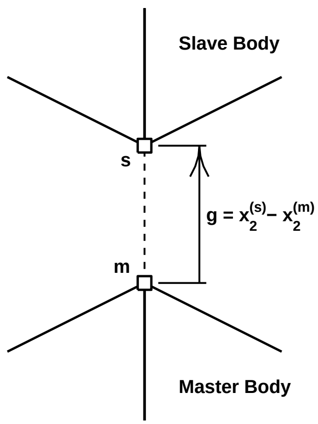
- Introduces new unknown (λ) for each contact pair.

- Equations **indefinite** (zero multiplier diagonal).

1985: Solution Methods

- **Perturbed lagrangian** form of solution

$$\Pi_c = \lambda\, g - \frac{1}{2\,\kappa}\lambda^2$$



- Variational equation

$$\delta\Pi_c = \left[\ \delta u_2^{(s)} \quad \delta u_2^{(m)} \quad \delta\lambda\ \right] \left\{ \begin{array}{c} \lambda \\ -\lambda \\ g - \frac{1}{\kappa}\lambda \end{array} \right\}$$

- Solution: $\lambda = \kappa\, g$ where κ **constraint parameter** to prevent penetration (penalty).

1985: Solution Methods

- Linearization produces tangent matrix

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ 1 & -1 & -1/\kappa \end{bmatrix} \begin{Bmatrix} du_2^{(s)} \\ du_2^{(m)} \\ d\lambda \end{Bmatrix} = \begin{Bmatrix} -\lambda \\ \lambda \\ -g + \frac{1}{\kappa}\lambda \end{Bmatrix}$$

- Eliminate λ and $d\lambda$ gives reduced tangent

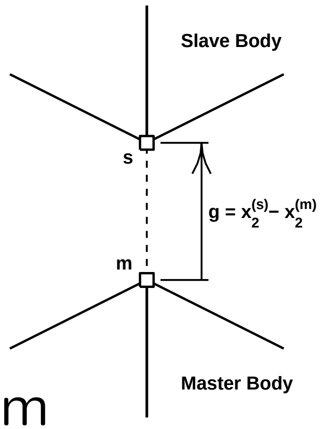
$$\begin{bmatrix} \kappa & -\kappa \\ -\kappa & \kappa \end{bmatrix} \begin{Bmatrix} du_2^{(s)} \\ du_2^{(m)} \end{Bmatrix} = \begin{Bmatrix} -\kappa\, g \\ \kappa\, g \end{Bmatrix}$$

- **Final gap non-zero** - depends on value of κ.

1985: Solution Methods

- **Perturbed tangent** form (Lim & T, 2001)

- Lagrange multiplier variational equation



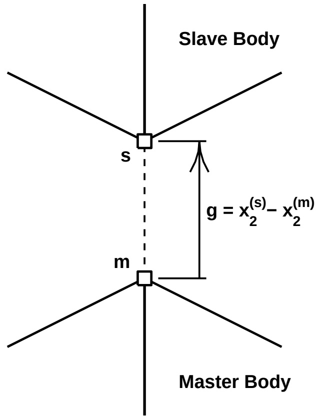
$$\delta \Pi_c = \begin{bmatrix} \delta u_2^{(s)} & \delta u_2^{(m)} & \delta \lambda \end{bmatrix} \begin{Bmatrix} \lambda \\ -\lambda \\ g \end{Bmatrix}$$

- Linearized form for perturbed lagrangian form

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ 1 & -1 & -1/\kappa \end{bmatrix} \begin{Bmatrix} du_2^{(s)} \\ du_2^{(m)} \\ d\lambda \end{Bmatrix} = \begin{Bmatrix} -\lambda \\ \lambda \\ -g \end{Bmatrix}$$

- Not consistently linearized form (non-zero diagonal for λ).

1985: Solution Methods



- **Penalty function** form

$$\Pi = \frac{1}{2} \kappa \, g^2$$

where κ is penalty parameter.

- Matrix equation for nodes given by

$$\begin{bmatrix} \kappa & -\kappa \\ -\kappa & \kappa \end{bmatrix} \begin{Bmatrix} du_2^{(s)} \\ du_2^{(m)} \end{Bmatrix} = \begin{Bmatrix} -\kappa \, g \\ \kappa \, g \end{Bmatrix}$$

- N.B. Not always same as perturbed lagrangian form.

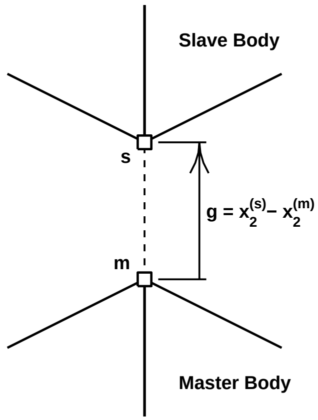
- Avoids indefinite equation of Lagrange multiplier approach.

1985: Solution Methods

- **Augmented Lagrangian** penalty form

- Mix of penalty and Lagrange multiplier

$$\begin{bmatrix} \kappa & -\kappa \\ -\kappa & \kappa \end{bmatrix} \left\{ \begin{array}{c} du_2^{(s)} \\ du_2^{(m)} \end{array} \right\} = \left\{ \begin{array}{r} -\lambda_k - \kappa\, g \\ \lambda_k + \kappa\, g \end{array} \right\}$$



$$g = x_2^{(s)} - x_2^{(m)}$$

- Update to 'Lagrange multiplier' computed using

$$\lambda_{k+1} = \lambda_k + \kappa\, g$$

- Update after each Newton iteration or in new loop.

1985: Solution Methods

- **Augmented Lagrangian** perturbed form

$$\Pi_c = (\lambda + \lambda_k)\, g - \frac{1}{2\,\kappa} \lambda^2$$

- Mix with perturbed Lagrangian form:

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ 1 & -1 & -1/\kappa \end{bmatrix} \begin{Bmatrix} du_2^{(s)} \\ du_2^{(m)} \\ d\lambda \end{Bmatrix} = \begin{Bmatrix} -(\lambda + \lambda_k) \\ (\lambda + \lambda_k) \\ -g + \frac{1}{\kappa}\lambda \end{Bmatrix}$$

where Uzawa update $\lambda_{k+1} = \lambda_k + \kappa\, g$ used.

- **Method of choice to program**: Can reduce to all other forms.

1988: Friction by Perturbed Lagrangian

- Friction - **Coulomb model** simplest form

$$\lambda_t \leq \mu \mid \lambda_n \mid$$

- Stick: If $\mid \lambda_t \mid < \mu \mid \lambda_n \mid$ set $g_t = 0$

- Variational equation

$$\delta \Pi_c = \delta \lambda_n (g_n - \frac{1}{\kappa_n} \lambda_n) + \delta g_n \, \lambda_n + \delta \lambda_t (g_t - \frac{1}{\kappa_t} \lambda_t) + \delta g_t \, \lambda_t$$

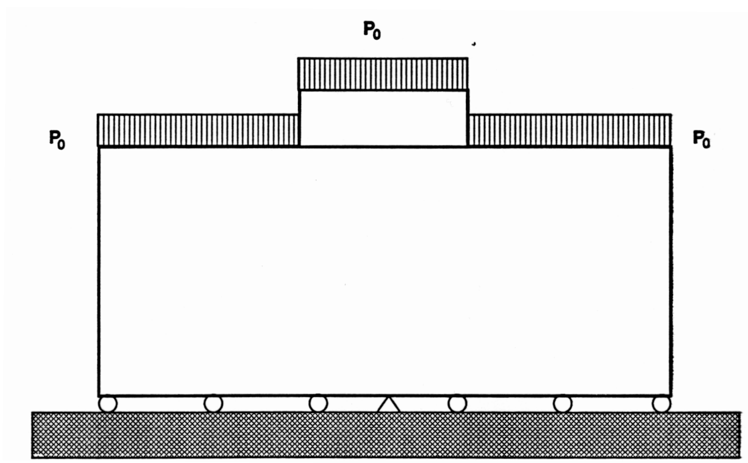
  - Slip : If $\mid \lambda_t \mid = \mu \mid \lambda_n \mid$ gives $g_t \neq 0$

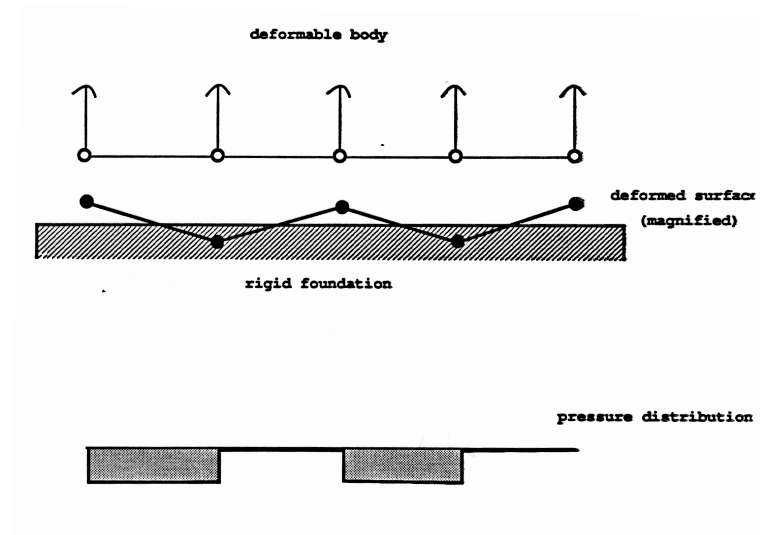
$$\delta \Pi_c = \delta \lambda_n (g_n - \frac{1}{\kappa_n} \lambda_n) + \delta g_n \, \lambda_n + \delta \lambda_t (g_t - \frac{1}{\kappa_t} \lambda_t) + \delta g_t \, [\mu \mid \lambda_n \mid \mathrm{sign}(gt)]$$

  - Yields **unsymmetric tangent**(Ju & T, 1988).
  - Augmented form can make symmetric - Laursen & Simo (1993).

1991: The Contact Patch Test

- Contact patch: **Consistency** (constant stress) & **Stability** test (Papadopoulos & T, 1991).





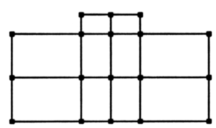
1991: The Contact Patch Test

PATCH TEST

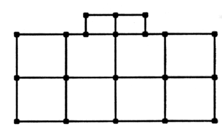
- Contact patch test used to assess consistency

Patch Test Results							
Pressure	Element Meshes						
Constant	A	B-1p	B-2p	C-1p	C -2p	D	E
Result	p	p/f	p	f	p	p	f

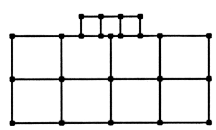
Results: p = pass; f = fail; p/f = master surface dependent.



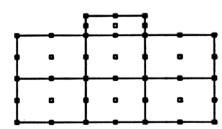
Mesh A



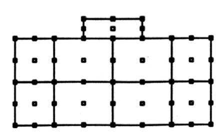
Mesh B



Mesh C



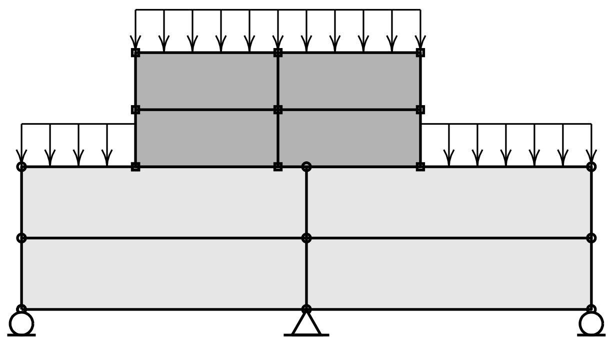
Mesh D

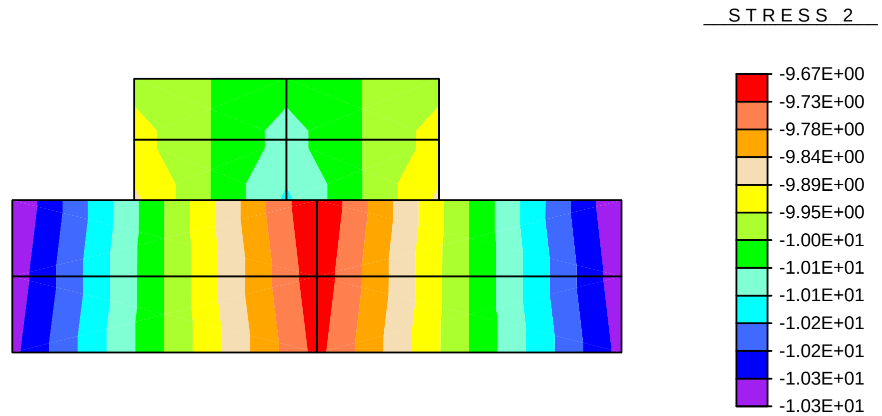


Mesh E

1991: The Contact Patch Test

- Example: Node-Surface: 1-pass





(a) Mesh and loading (b) σ_{22} stress result

- Test **fails** for node to segment treatment.
- Two pass switch master/slave: Penalty passes (**penetrate**). Augmented lagrangian or perturbed tangent fails test.

FEM Node-to-Surface Contact Analysis

- All previous schemes have limitations:
  - Valid only for low order elements (3-node triangles & 4-node quadrilaterals in 2-D; 4-node tets to 8-node bricks in 3-D).
  - Fails **contact patch test** unless two-pass scheme used.
  - Two-pass works in penalty form - fails **stability test** for Lagrange multiplier.

2002-2004: Mortar Methods for Contact & Ties

- Contact interface requires addition of term

$$\Pi_c = \int_{\Gamma_s} \boldsymbol{\lambda}^T \left(\mathbf{x}^{(m)} - \mathbf{x}^{(s)} \right) \, \mathrm{d}\Gamma$$

where $\mathbf{x}^{(m)}$ and $\mathbf{x}^{(s)}$ deformed positions of master & slave.

- Integrals carried out by quadrature on **sub-segments**.

- Use interpolations

$$\mathbf{x}^{(m)} = N_a(\boldsymbol{\xi}) \, \tilde{\mathbf{x}}_a^{(m)}(t) \ ; \ \mathbf{x}^{(s)} = N_b(\boldsymbol{\xi}) \, \tilde{\mathbf{x}}_b^{(s)}(t) \ ; \ \boldsymbol{\lambda} = \hat{N}_c(\boldsymbol{\xi}) \, \tilde{\boldsymbol{\lambda}}_c(t)$$

where N_a, N_b usual shape functions and $\hat{N}_c$ multiplier ones.

2002-2004: Mortar Methods for Contact & Ties

- Options for $\widehat{N}_c$:
  - Delta function $\widehat{N}_c = \delta(\boldsymbol{\xi} - \boldsymbol{\xi}_c)$ gives **node to surface**.
  - Slave shape function $\widehat{N}_c = \delta_{ca}\, N_a(\xi)$ gives **standard mortar method**.
  - Dual of slave shape function (e.g., linear case)

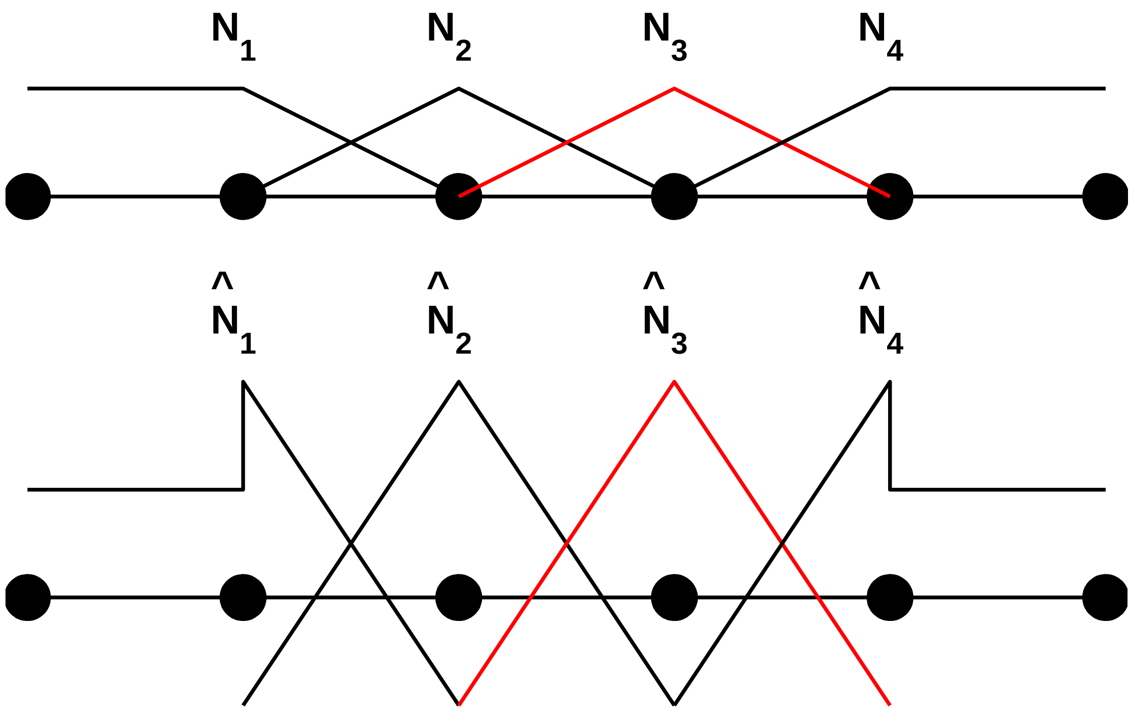
    $$\widehat{N}_c = \alpha_1\, N_1 + \alpha_2\, N_2$$

    gives **dual mortar method**.

- Dual mortar method has advantages.

2002-2004: Mortar Methods for Contact & Ties

- Two-dimensional mortar functions and duals



2002-2004: Mortar Methods for Contact & Ties

- Two-dimensional mortar shape functions and duals
  - Dual satisfies

$$\int_h \widehat{N}_a N_b \, \mathsf{d}\Gamma = \delta_{ab} \int_h N_a \, \mathsf{d}\Gamma$$

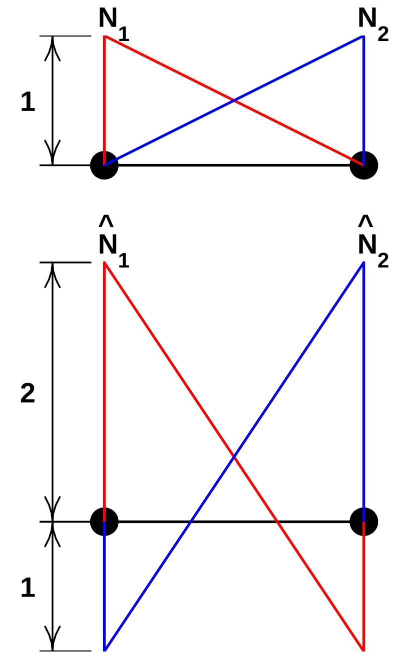
  - Assume linear functions

$$\widehat{N}_a = \alpha_1 \, N_1 + \alpha_2 \, N_2$$

gives for $\widehat{N}_1$

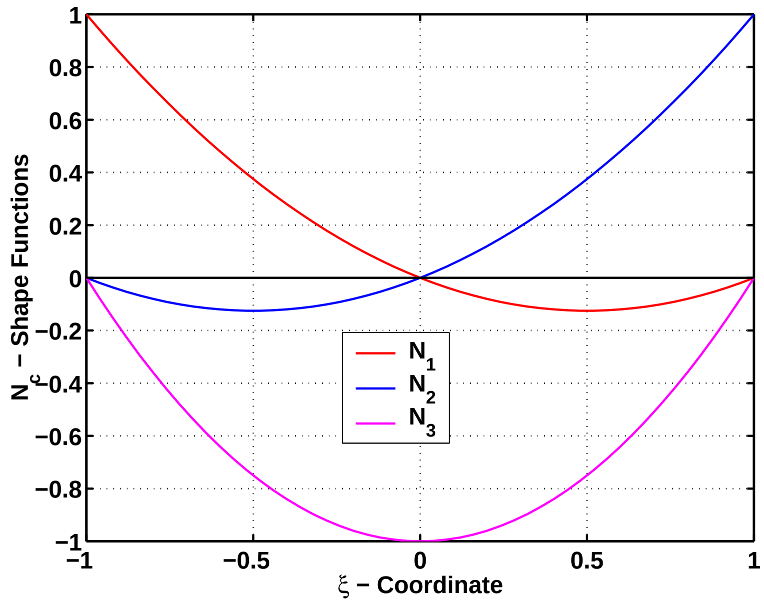
$$\frac{h}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \frac{h}{2} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Solution: $\alpha_1 = 2$, $\alpha_2 = -1$.

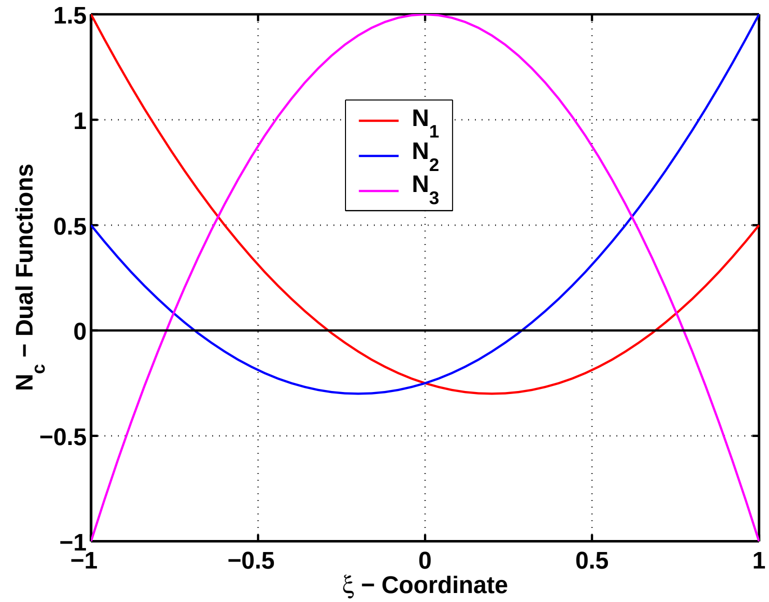


2002-2004: Mortar Methods for Contact & Ties

- Quadratic functions:



(a) Shape functions



(b) Dual Functions

2002-2004: Mortar Methods for Contact & Ties

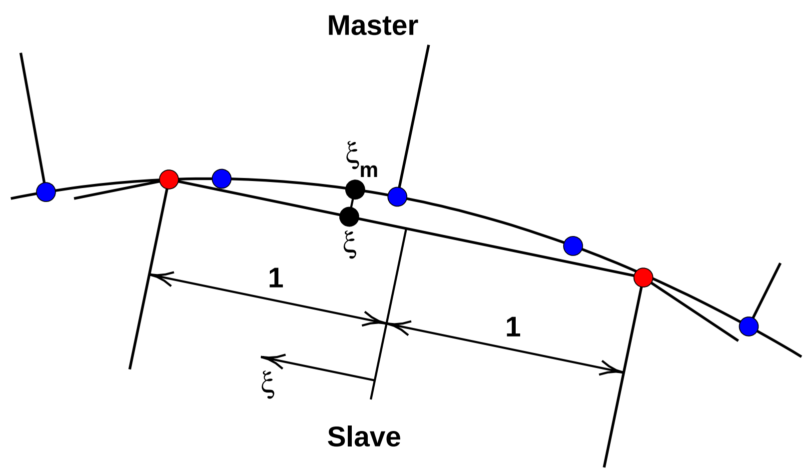
- Substitution of Discretization in

$$\Pi_c = \int_{\Gamma} \boldsymbol{\lambda}^T \left(\mathbf{x}^{(m)} - \mathbf{x}^{(s)} \right) \, \mathrm{d}\Gamma$$

and integrating on **slave** facets

$$\Pi_c = \sum_s \int_{\Gamma_s} \tilde{\boldsymbol{\lambda}}_c^T \widehat{N}_c^s(\xi) \left[N_a^s(\xi) \tilde{\mathbf{x}}_a^s - N_b^m(\xi_m) \tilde{\mathbf{x}}_b^m \right] \, \mathrm{d}\Gamma$$

where ξ for slave and ξ_m projected master point.

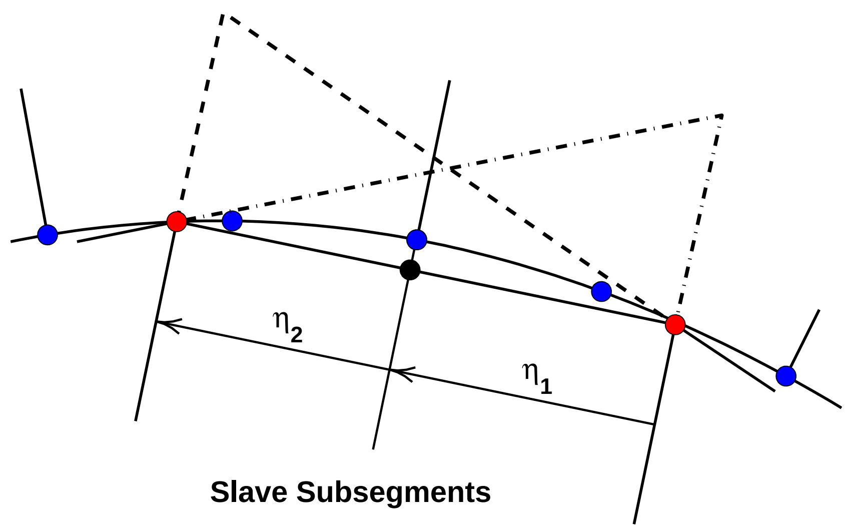


2002-2004: Mortar Methods for Contact & Ties

- Accurate quadrature uses segments

$$\int_{\Gamma_s} f(\xi)\,\mathrm{d}\xi = \sum_m \int_{\Gamma_{ms}} \hat{f}(\eta)\,\mathrm{d}\eta$$

  with segment $-1 \le \eta \le 1$.



- Gauss-Legendre quadrature needs 2-points/segment for linear-linear sides; 3-points/segment for quadratic-quadratic sides.

2002-2004: Mortar Methods for Contact & Ties

- Evaluation of segment integrals gives

$$\Pi_s = \tilde{\boldsymbol{\lambda}}_c^T \left[\mathbf{G}_{ca}^s \, \tilde{\mathbf{x}}_a^s - \mathbf{G}_{cb}^m \, \tilde{\mathbf{x}}_b^m\right]$$

  where $\Pi_c = \sum_s \Pi_s$ and

$$\mathbf{G}_{ca}^s = \int_{\Gamma_s} \widehat{N}_c(\xi) \, N_a(\xi) \, \mathrm{d}\Gamma \, \mathbf{I}$$
$$\mathbf{G}_{cb}^m = \int_{\Gamma_s} \widehat{N}_c(\xi) \, N_b(\xi_m) \, \mathrm{d}\Gamma \, \mathbf{I}$$

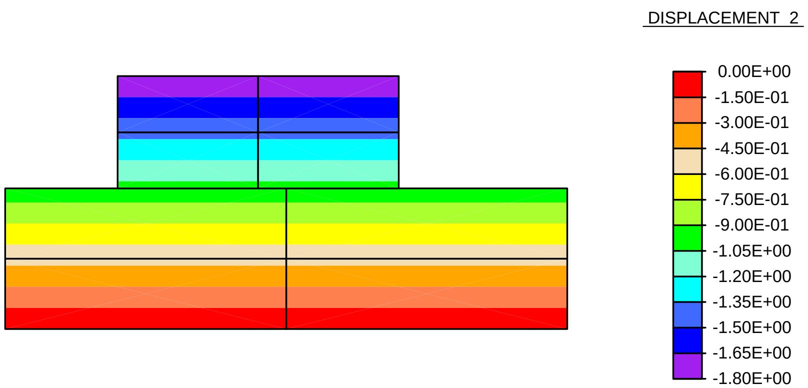
  where $\mathbf{I}$ are $ndim \times ndim$.

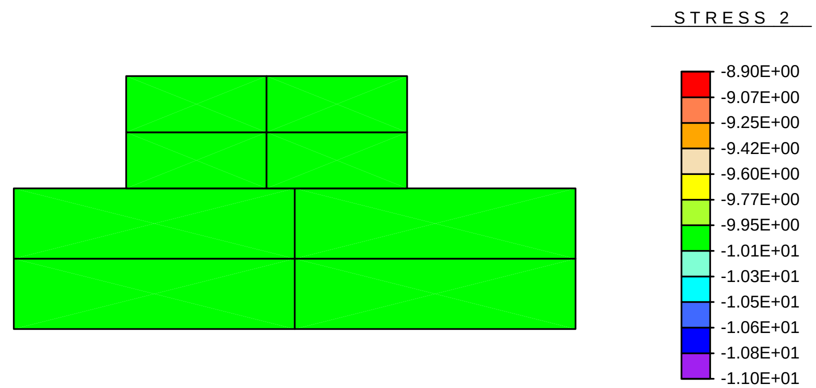
- For <u>dual mortar</u>: $\mathbf{G}_{ca}^s$ is **diagonal**.

- Conserve linear and angular momenta (Puso & Laursen, 2003).

2002-2004: Mortar Methods for Tied Surfaces

- Surface to surface tied interface - 4 node elements.



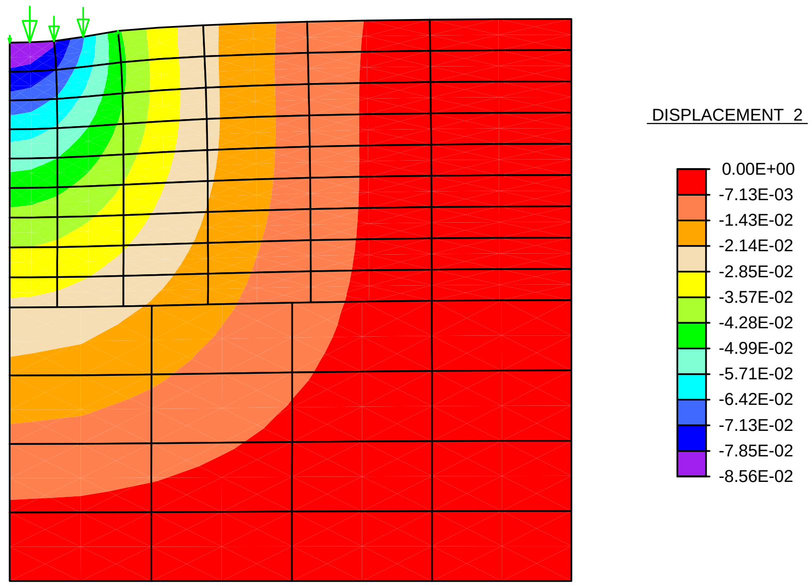


(a) Vertical displacement

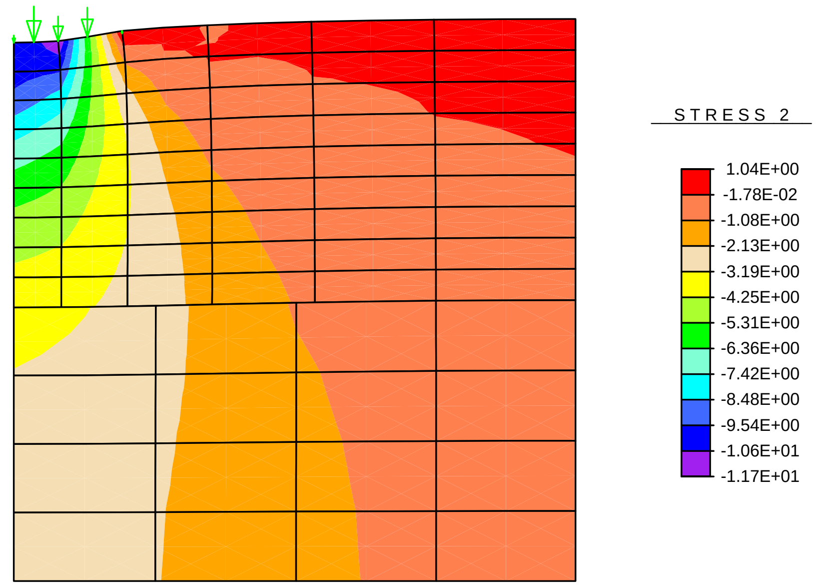
(b) Vertical stress

2002-2004: Mortar Methods for Tied Surfaces

- Surface load on layer - 9 node elements



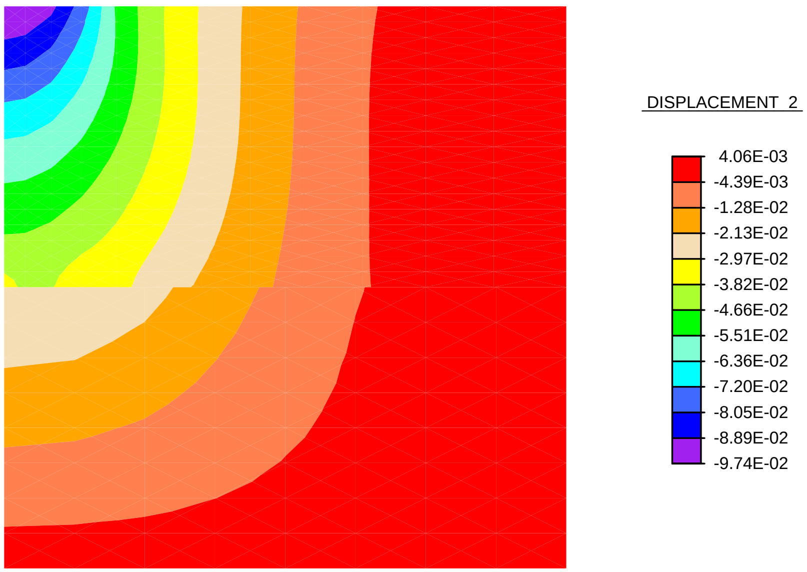
(a) u_2 displacement



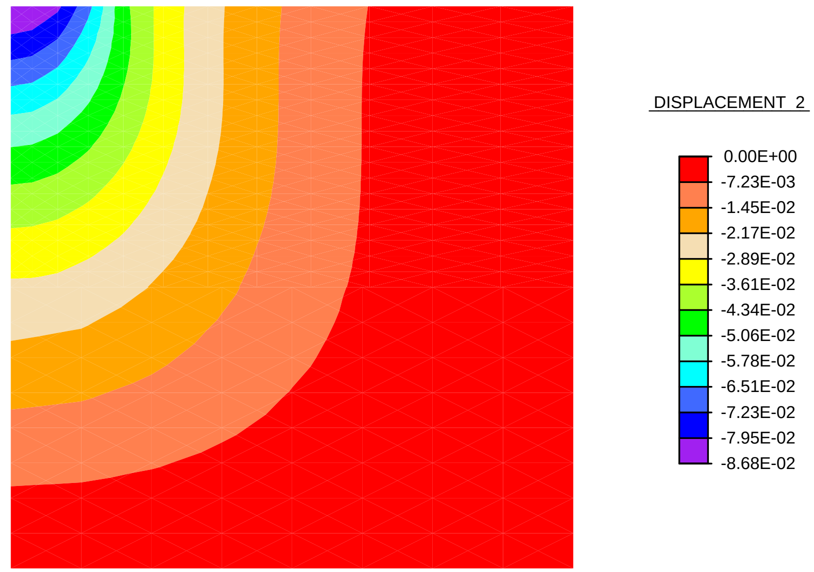
(b) σ_{22} stress

2002-2004: Mortar Methods for Tied Surfaces

- Displacement for surface load on layer - 4 node elements



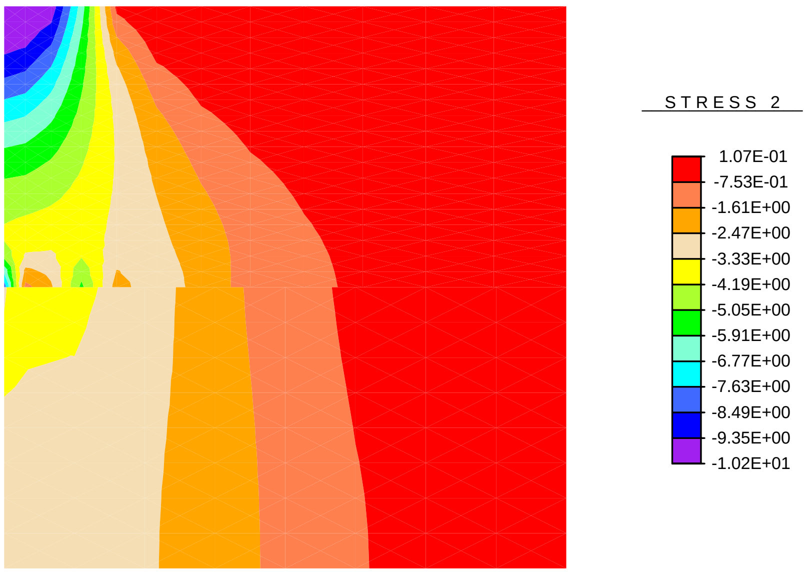
(a) Node to Surface



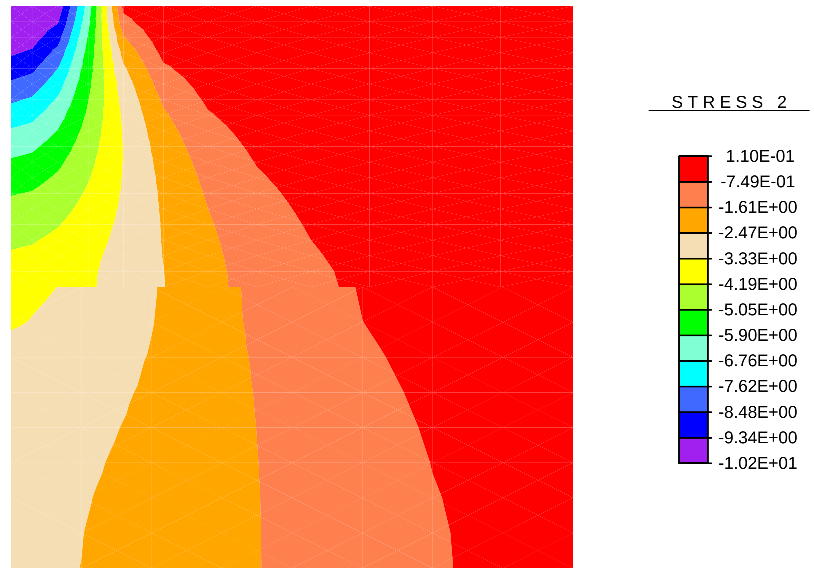
(b) Mortar Method

2002-2004: Mortar Methods for Tied Surfaces

- Stress for surface load on layer - 4 node elements



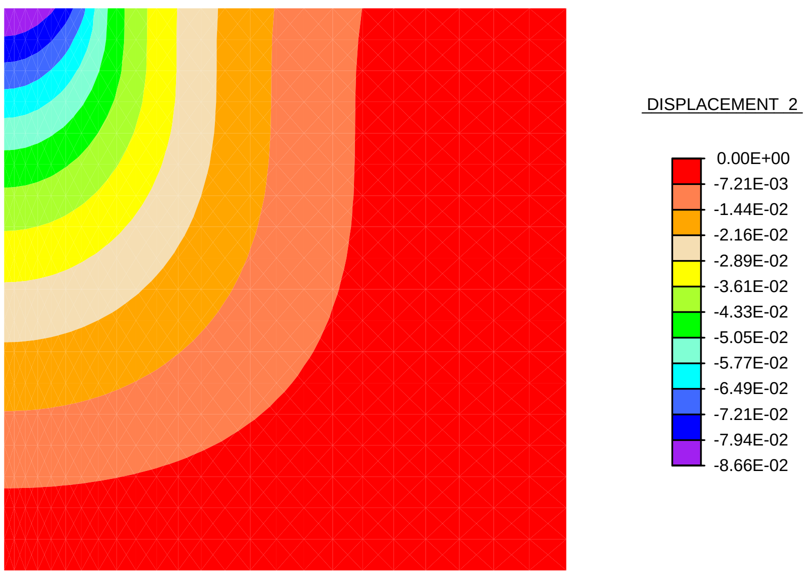
(a) Node to Surface



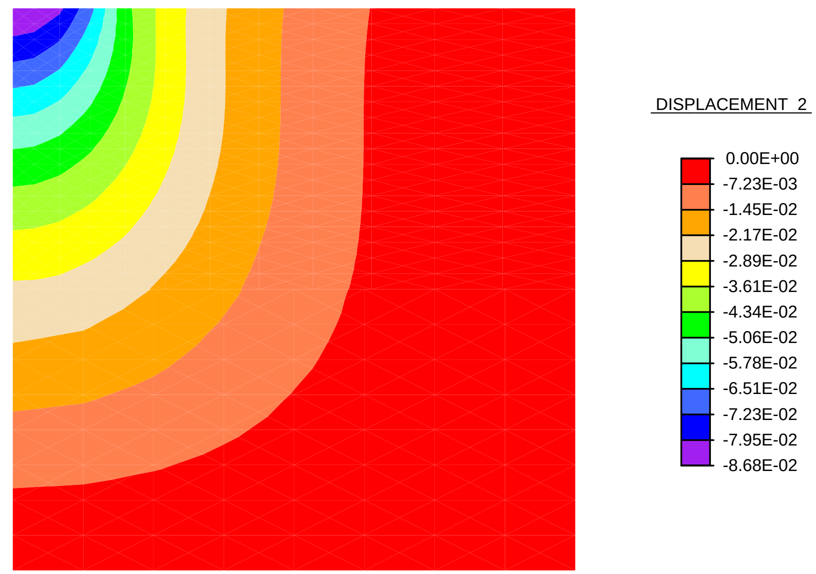
(b) Mortar Method

2002-2004: Mortar Methods for Tied Surfaces

- Displacement for surface load on layer - 4 node elements



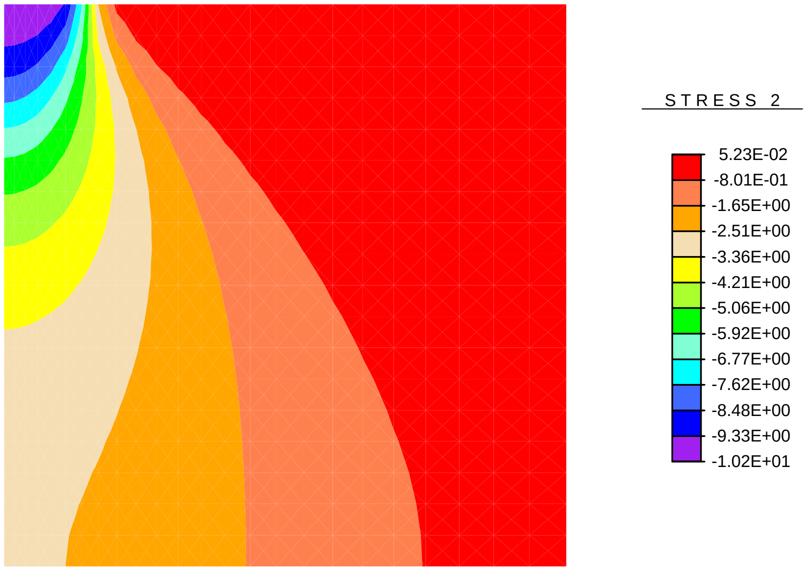
(a) No Interface



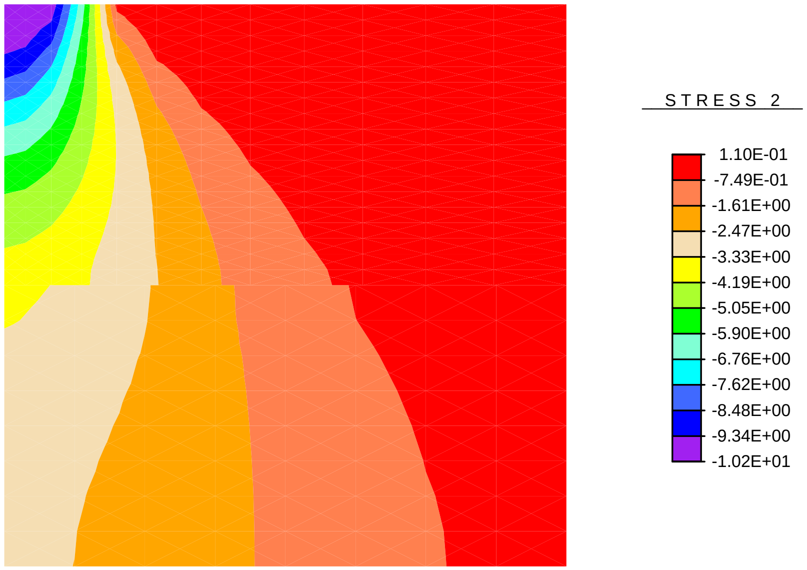
(b) Mortar Method

2002-2004: Mortar Methods for Tied Surfaces

- Stress for surface load on layer - 4 node elements



(a) No Interface



(b) Mortar Method

2002-2004: Mortar Methods for Tied Surfaces

- Discussed only **tied interface** treatment.
- Implentation for full 2-d & 3-d contact in progress.
- Best to date observed is **NIKE** at LLNL by M.A. Puso.
- Gives good results without surface smoothing.

Summary: Contact Analysis

- Lecture discussed:
  - Foundations for contact analysis, including transient behavior for impact/release. All extensions of contributions by TJRH!
  - Solution strategies for contact analysis (Lagrange multiplier, penalty, etc.)
  - Contact patch test requirements – consistency & stability.
  - Spatial discretization methods (node-node; node-surface; surface-surface).
  - Mortar (standard & dual) methods for surface-surface treatment.

Happy 60th, Tom!