PROBLEM SETS:

Fundamentals of Transportation and Traffic Operations

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Fundamentals of Transportation and Traffic Operations
Problem Sets

Preface

These problem sets comprise a supplement to *Fundamentals of Transportation and Traffic Operations* (C. Daganzo, Pergamon, 1997). Academicians can also obtain a companion set of solutions by writing to "Institute of Transportation Studies, Publications Office, 109 McLaughlin Hall, University of California, Berkeley, CA 94720" or by sending e-mail to its@its.berkeley.edu.

This collection of problems is a result of the author's teaching experience over numerous years. Some of the problems have been created by G. F. Newell, and this is acknowledged on a case by case basis. The majority of the problems have been used in the classroom. Others have been added recently to supplement the newest portions of *Fundamentals*. Organized by chapter, the exercises range in difficulty from elementary, as would be appropriate for undergraduates, to advanced as may be appropriate for a Ph.D. qualifying examination. The latter are marked by an asterisk. Most exercises are intermediate in difficulty, however.

Errata for *Fundamentals of Transportation and Traffic Operations* are included in paper copies of this document, and are also available on the publications section of Carlos Daganzo's world wide web home page at http://www.ce.berkeley.edu/~daganzo.

The compilation would not have been possible without the help of the three authors of the companion solution set. Robert Bertini (U.C. Berkeley) managed the overall preparation of the problem sets and solutions. He improved the wording, graphics and notation of many of the problems, and facilitated the internet availability. David Lovell (University of Maryland) provided improved problem statements for several significant problems and Wei Lin (Virginia Polytechnic Institute and State University) also provided helpful comments on several problem statements. Thanks also go to Ms. Alison Andreas for her excellent word processing.

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Fundamentals of Transportation and Traffic Operations
Chapter 1 Problems

Problem 1.1

Consider that the maximum allowable value of the jerk for a train with standing passengers is $\tilde{j}$ (m/s^3), its maximum acceleration is $\tilde{a}$ (m/s^2), and its maximum velocity is $\tilde{v}$ (m/s). First, find an expression describing the minimum distance d_{min} between stops such that $\tilde{v}$ is just developed. Then, determine the time, t, needed for the train to travel any given distance, d ($d > d_{min}$), from a complete stop to a complete stop. Assume that the maximum acceleration can be considered to be independent of speed.

Problem 1.2

If the typical acceleration/deceleration of a car moving up in position in a queue at a stop sign is a = 0.1g, determine the time that it takes for a car to advance from the second position into the first. Assume a reasonable reaction time and car spacing for the vehicle in question and discuss how this calculation may relate to the maximum number of vehicles that may advance past a stop sign in 1 minute. How do you think variations in a across vehicles would affect this number?

Problem 1.3

A train descending a 2.0 percent grade at 40 mi/hr applies the brakes at t = 0. If the friction factor that is developed is 0.05, then find:

(a) The speed of the train after it has traveled 1,000 ft.
(Note: distances along the track and along its horizontal projection are virtually the same because of the small angle.)

(b) The elapsed time after the train has traveled 1000 ft.

Problem 1.4

A railroad car with an initial speed v_o is pushed over the hump of a railroad yard that is h feet above its final track. The car should be retarded so as to reach its final destination *x* ft. away from the hump also at $v = v_o$. Because the speeds reached are low, the acceleration of the car is assumed to be the sum of a constant, $-fg$, (independent of speed) that captures friction losses and a term that is proportional to the grade $-gs$. If the car weighs *W* Newtons, determine the magnitude of the retarding force that needs to be applied while the wheels of the car travel over a retarder of length *L* if the retarder is located at the level of the final track, so as to achieve the desired effect.

Problem 1.5

This problem examines the motion of an automobile traveling on a crest vertical curve with the following profile:

$$y = 100 + 0.10x - 0.0005x^2,$$

where *y* and *x* represent respectively the roadway elevation (in feet) and the horizontal distance (also in feet). Assume that for the small grades that occur in roadways, horizontal distances are nearly equal to those actually traveled. The automobile sets the brakes when $x = 0$, while traveling at 45 mi/hr. The friction factor is 0.3, and air resistance is negligible. Then:

(a) Find the stopping distance ignoring the centrifugal acceleration.

(b) Repeat the calculations considering the levitating effect of the acceleration. (Hint: Solve this part numerically with a spreadsheet).

Problem 1.6

The state of Nevada allows 3-trailer truck combinations, while neighboring states allow a maximum of 2 trailers per truck. Sketch a (t,x) diagram that depicts the trajectories of several truck cabs and trailers, assuming that every trailer-truck combination is stopped at the borders to ensure that every combination always carries the maximum possible number of trailers.

If westbound 2-trailer trucks arrive regularly at the Nevada/Utah border every 15 minutes and eastbound 2-trailer trucks arrive every 20 minutes at the Nevada/California border:

(a) Use the (t,x) diagram to sketch the trajectories of cabs and trailers for one day, assuming that it takes 8 hours to cross Nevada and 30 minutes to decouple or re-couple a trailer-truck combination. You should use an inset to depict in detail what happens at each border.

(b) Superimpose on your picture the trajectories of the drivers for a strategy that would use them efficiently.

Problem 1.7

Devise a two-way bus service scheme on the (regional) route {A→ B→ C:C→ B→ A} that will have regular departures from A, but buses continuing on from B to C will depart less frequently. Some buses will be turned back at B and will return to A without continuing on to C. Passengers from A to C should not transfer at B but they may be delayed at B.

If the target service frequencies on A ↔ B are 12 buses/hr in each direction, and on B ↔ C, 8 buses/hr, and if the trip times from A→ B, B → A, C → B, and B → A are each 30 minutes, then depict a set of bus trajectories that will meet these requirements with regular headways on all legs. You need to determine how much to delay each bus at B, and if a bus just came from A, whether to turn it around at B.

What size of bus fleet is required to accommodate your scheme?

Problem 1.8 *(Courtesy of G. Newell)*

If all trucks travel at speed v', cars at speed v, and the fraction of vehicles passing a stationary observer that are trucks is p, what fraction of vehicles would be trucks on a photograph? If the length of a truck is twice that of a car, how many car lengths is the average vehicle on a photograph?

Evaluate these for $v' = 50$ km/hr, $v = 80$ km/hr, and $p = 0.7$.

Problem 1.9 *(Courtesy of G. Newell)*

If the speed limit on a highway network is changed so that the average speed drops from v to v' (100 km/hr to 80 km/hr, for example), what happens to q and k if people continue to make the same trips every day?

Problem 1.10 *(Courtesy of G. Newell)*

From two consecutive film frames of traffic flow along a road, one observes that there are ten cars per kilometer having zero velocity (they are parked), 20 cars per kilometer traveling at 10km/hr and 40 cars traveling at 20km/hr.

(a) Determine the space mean speed and the time mean speed of traffic.

(b) If individual cars can maintain these speeds even while passing other cars, how many cars will a driver traveling at 20km/hr pass while traveling one kilometer?

Problem 1.11 *(Courtesy of G. Newell)*

From a photograph one observes that on a level section of highway 10% of the vehicles are trucks, 90% are cars, and that there are 50 vehicles per mile of highway. The trucks travel at 40 mi/hr, and the cars at 50 mi/hr. This highway also has a section with a steep grade on which the speed of the trucks drops to 20 mi/hr, and the speed of the cars to 40 mi/hr. No vehicles enter the observed sections of highways (except at the ends), and the flows are (nearly) stationary.

Determine:

(a) The flow of vehicles on the level section.

(b) The density of vehicles on the grade.

(c) The percent of trucks on the grade as seen on a photograph.

(d) The percent of trucks as seen by a stationary observer on the grade.

Problem 1.12

A one way road is shared by buses and cars. Buses travel at speed v' and carry n' people. Cars travel at speed v and carry n people. The fraction of vehicles that are buses passing a stationary observer is p. Answer the following questions:

(a) What fraction of the vehicles on a photograph would be buses?

(b) What is the average vehicle occupancy seen by a stationary observer?

(c) What is the average vehicle occupancy seen on a photograph? Evaluate

both occupancy averages for $v' = 50$ km/hr, $v = 80$ km/hr, $n = 1$, $n' = 20$ and $p = 0.2$.

(d) Consider the numerical data of part (c). If a bus emits 2 ½ times the pollutants emitted by a car per unit time, what fraction of the total pollution is generated by the cars along one kilometer of road in one hour?

Problem 1.13*

The distribution of vehicle speeds measured by a stationary observer is uniform in the interval $[v_{\min}, v_{\max}]$. The speed limit is $\tilde{v} \in (v_{\min}, v_{\max})$. Determine the fraction of speeding vehicles seen by a police car traveling at speed v_0. Plot for $-\infty \leq v_0 \leq \infty$. Discuss.

Problem 1.14

The wheel base of a bicycle is D and the friction factor is f. The center of gravity of the bicycle-rider assembly is h distance units above the ground, and is centered between the wheels.

Find the maximum deceleration that can be achieved by using the rear brake alone. Do not forget to include the inertial pseudo-forces that cause the normal reaction between the ground and the wheels to be biased toward the front under deceleration.

Fundamentals of Transportation and Traffic Operations
Chapter 2 Problems

Problem 2.1

Sketch a cumulative flow diagram that represents the growth and dissipation of a rush hour period at a toll bridge with time-independent capacity.

(a) Identify on the diagram: the arrival curve $A(t)$, the maximum queue, the total delay W, the duration of the saturated period, T, and the service rate μ.

(b) Determine the change in delay if the service rate is increased by a small amount $\Delta\mu$ during the last half of the saturated period.

(c) Determine the change in the delay if the service rate is increased by an amount $\Delta\mu/2$ for the whole duration of the period (involving the same number of employee hours) and/or by an amount $\Delta\mu$ for the first half of the period. Discuss.

Problem 2.2

A ramp meter is being considered at an entrance to a freeway. Currently, rush hour traffic arrives at the on-ramp at a rate q_1 from time $t = 0$ to time $t = t^*$. After $t = t^*$, vehicles arrive at a (lesser) rate q_2.

(a) Assuming that drivers will not change their trips, draw and label an input-output diagram showing a metering (i.e., departure) rate of μ ($q_2 < \mu < q_1$). Label the maximum delay experienced by any vehicle (w_{max}).

(b) If an alternate route is available to drivers, and it is known that they will take this route if their expected delay at the ramp meter is greater than $w_{max}/2$, add this new scenario to your diagram. Now, show graphically the following:

(1) The number of vehicles which will divert.

(2) How much earlier the queue will dissipate (compared to part (a)).

Problem 2.3

A vehicle (*A*) traveling on a freeway joins a ½-mile queue that contains 100 vehicles at a time t = 0 min. Vehicles in this queue pass through the bottleneck at a rate of 50 veh/min. When there is no queue, vehicles travel (in free flow) at a rate of 1 mile/min, provided that the flow satisfies: $q < q_{max}$ = 100 veh/min. Do the following:

(a) Determine the delay and the time in queue for our hypothetical vehicle.

(b) Determine the (average) density of vehicles in the queue.

(c) Plot a triangular flow-density relation for our freeway that will be consistent with the given data.

(d) If the (free) flow upstream of the bottleneck is 80 veh/min, determine the location of the end of the queue (in miles upstream of the bottleneck) 1 minute after the arrival of vehicle *A*. Solve this with the help of a picture, drawn to scale, as follows:

(1) Construct the virtual arrival curve, the bottleneck departure curve and the back-of-queue curve, starting with vehicle A.

(2) Identify on the picture the vehicle (*B*) that joins the queue at t = 1 min.

(3) Determine the distance in queue for such vehicle.

Problem 2.4

Suppose that the two approaches to a freeway merge, labeled *1* and *2*, can send a combined maximum flow μ = 4,000 veh/hr through the merge without queues forming. Suppose as well that if both approaches are queued, vehicles enter the merge in a 3:1 ratio; i.e., three vehicles from approach *1* for each vehicle from approach *2*. Furthermore, if only approach i is queued then the vehicles from this approach are assumed to enter in a higher ratio (with μ = 4,000 veh/hr). Then describe what would happen if there are no queues and two surges of traffic, with 3,500 veh/hr on approach 1 and 700 veh/hr on approach 2, hit the merge simultaneously. Use a diagram and explain.

Problem 2.5

A minor street forms a tee-intersection with a major through-street. The minor street traffic must stop at a stop sign. Traffic arriving at the stop sign must either turn right (at rate μ_1) or turn left (at rate $\mu_2 \ll \mu_1$). If the right-turning and left-turning virtual arrival curves, $V_1(t)$ and $V_2(t)$, are given, determine the total delay.

Problem 2.6

This is an example of a time-dependent O-D table estimation problem in a congested network. If curves D_1 and D_2 are observed downstream of a congested diverge or intersection, and an upstream curve $A(t)$ is given that is consistent with D_1 and D_2, then determine curves $A_1(t)$ and $A_2(t)$ assuming a FIFO discipline.

Problem 2.7*

Suppose that the service times of a FIFO freeway bottleneck when vehicle n is discharging depends on its waiting time, w_n, in the following way:

$$s(n) = s_1 + (s_0 - s_1)\ e^{-w_n/\tilde{w}} \quad (s_1 > s_0 > 0)$$

where s_1 and s_0 are the maximum and minimum service times and $\tilde{w}$ is a *characteristic* waiting time. (This type of model has been proposed to explain the drop in capacity observed in some Japanese motorways subject to severe congestion.).

(a) Assuming such a system reaches an equilibrium, what is the equilibrium queue length?

Suppose now that at the onset of the rush hour prior to which there is no queueing and no delay, the (virtual) arrival curve at the bottleneck jumps suddenly by n_0 vehicles and then increases steadily with a slope

$$\lambda_0 \in \left(\frac{1}{s_1}, \frac{1}{s_0}\right)$$

(b) Sketch a departure curve and the growth of delay during the rush hour, as you would expect it to happen. Would you expect the system to reach an equilibrium? Why? Would the value of λ_0 matter?

(c) Derive an expression for the critical value of λ_0 (λ_{crit}) beyond which the system cannot reach an equilibrium.

(d) Discuss the dependence of λ_{crit} on n_0 and the effect that changes in n_0 have on our system (with a fixed λ_0) for the duration of the rush hour. Suppose for example that n_0 varies from day to day.

Problem 2.8

Suppose that the customer cumulative (virtual) arrival curve at a bottleneck is of the form:

$$V(t) = \text{middle}\ (0, (N_i\ /T)\ t, N_i) \quad \text{for } N_i, T > 0.$$

where N_i is the total number of customers served during day i. If the bottleneck serves customers at a uniform rate (μ) every day, and N_i varies across days so that 50% of the days ($N_i\ /T$) = 2μ and 50% of the days ($N_i\ /T$) = 3μ, determine:

(a) The total delay accumulated in the system after N days (N large), the average delay per day, and the average delay per customer over all the customers.

(b) V_{avg}, D_{avg}, the area between these curves, and the average delay per customers one would estimate if all days were like the average day. Compare with (a).

(c) Derive the average across days of the average delay per customer $(\overline{w})_{avg}$, and show that it is different from (a) and (b). Explain why.

Problem 2.9

The (t,x) trajectories of N vehicles (j=0,...,N) are given by the following equation:

$$x_j(t) = \min\ \{4t-4j,\ \tfrac{1}{2}t-j+10\},\ j=0,...,N$$

(a) Draw a (t,x) diagram showing several of these trajectories and discuss qualitatively what would be happening (label some of the trajectories).

(b) Draw cumulative plots of vehicle number (j) vs. t at the locations x=0 and x=10 of two observers.

(c) Calculate the total trip time experienced by vehicles 0 to 10 between x=0 and x=10:

(1) From the diagram (a).

(2) From the diagram (b).

(d) Repeat (c) for the maximum number of vehicles between the observers

(e) Draw on sketch (b) the loci of points where vehicles are observed to change speeds.

Problem 2.10

Suppose that the two observers of Problem 2.9 record the following N-curves $A(t)$ and $D(t)$:

$A(t)$ = min $\{t, 2t\text{-}2\}$

$D(t)$ = min $\{t\text{-}1, 2t\text{-}8\}$

(a) Plot these curves and then a set of (t,x) vehicle trajectories that would be consistent with these data (use two families of parallel lines).

(b) Describe qualitatively what you think is happening, and based on your conjecture:

(c) Plot $N(t,x)$ as a function of x for t=5 from x=-30 to x=30.

Fundamentals of Transportation and Traffic Operations
Chapter 3 Problems

Problem 3.1 *(Original version courtesy of G. Newell)*

The cumulative number of cars, A, arriving at a toll plaza exhibits a daily peak such that A is approximately given by the following:

$A = \alpha_1 t$, if $0 < t <$ 7:00 hrs,

$A = 7\alpha_1 + \alpha_2\ (t-7)$, if 7:00 $< t <$ 9:00 hrs, and

$A = 7\alpha_1 + 2\alpha_2 + \alpha_1\ (t-9)$ if 9:00 $< t <$ 24:00 hrs,

where α_1 and α_2 are arrival flows at different times of the day (in cars/hr), and t denotes the time of day in hours. Then, if the cost of providing round-the-clock service, C (\$/day), is proportional to the toll plaza capacity, μ (cars/hr), $C=\beta\mu$, and if each customer hour of delay is valued at δ (\$/hr), find an expression for the capacity level that will minimize the total cost per day.

The above scenario would be more realistic if the capacity could be temporarily reduced during the off-peak period (with only a few booths being operated) to a value μ_1 : $\alpha_1 < \mu_1 < \mu$. We assume that this is only done when there is no queue. Repeat the exercise if the savings from operating at the lower level for T hours are $\beta'(\mu - \mu_1)\ T$ \$/day $(24\beta' < \beta)$; i.e. if the daily cost is:

$$C = \beta\mu - \beta'(\mu - \mu_1)\ T.$$

(Hint: find first the best μ_1 and T for any given μ.)

Problem 3.2

A corridor of length L is to be served by bus transportation during the evening commute (all trips start at x=0 and end somewhere in the interval $(0, L)$). The demand is uniform in this interval, with density l (pax/day/mile). Because the corridor is too long, it has to be served by two separate routes, i=1 operating between x=0 and $x=d_1$

with headways H_1, and i=2 carrying passengers to $x \in (d_1, L)$ with headways H_2.

The cost of operation is assumed to be directly proportional to the bus-miles traveled. Then:

(a) Write an expression for the total number of bus-miles traveled per unit time (e.g. day) as a function of our parameters.

(b) Explain why the total number of passenger-hours of delay incurred during boarding and the total number of passenger-hours of delay incurred during offloading per unit time (e.g. per day) should depend on the decision variables as follows:

Boarding delay per day = $a \sum_i H_i d_i$

Offloading delay per day = $b \sum_i H_i d_i^2$

Here we assume that $d_2 = L - d_1$.

Boarding delay is incurred out-of-vehicle. Assume that passenger boarding time is negligible compared to headways, therefore delay can be counted as the difference between a passenger's arrival time at the station and the departure time of the bus carrying that passenger. Offloading delay for any passenger is incurred in-vehicle, and represents the time that passenger waits on the bus while other passengers disembark. Assume that every alighting passenger delays the bus by g time units (i.e. assume only one passenger disembarks per stop, as in a suburban setting where passengers request stops by ringing the bell).

Interpret the coefficients α and β.

(c) Write a mathematical program (MP) to help the transit agency decide what to do if the agency has a fixed number of buses, which essentially limits the number of vehicle-miles traveled to a maximum amount, M. Assume the bus company would like to minimize its own costs, plus the delay costs incurred to the user. (This is a politically loaded endeavor, as determining the trade-off between the two would depend drastically on perspective. Assume that the cost per vehicle-mile of travel is reflected in a parameter d, with units of (vehicle-hours per day) per (vehicle-mile)).

(d) Classify the MP if the H_i are held constant (route design problem) and also if the d_i are held constant (scheduling problem).

(e) Classify the MP if *all* of the variables are decision variables.

Problem 3.3

Problem 3.2 has 5 parameters (L, M, l, g, and d) in its formulation and 4 decision variables. Is it possible to perform a change of variable that will eliminate all the parameters? If so, write the dimensionless version of the problem. If not, write a version that has the fewest possible free parameters.

Fundamentals of Transportation and Traffic Operations
Chapter 4 Problems

Problem 4.1

The flows of cars and trucks on a very long upgrade are 10 and 2 vehicles per minute respectively. Trucks travel at 0.8 mi/min, and cars at 1.0 mi/min when not trapped behind a truck (otherwise they travel at 0.8 mi/min). The system is stationary and passing is possible. If the time used to climb a 1 mile section of the grade averaged across all the vehicles observed over a long time is 1.1 min, determine:

(a) The density of vehicles on the upgrade.

(b) The average speed on the grade, taken across cars only.

(c) The proportion of time that each car spends behind trucks.

Problem 4.2

From two consecutive film frames of stationary traffic along some road, one observes that there are 20 cars per kilometer traveling at 100 km/hr and 30 cars per kilometer traveling at 120 km/hr.

(a) If individual cars can maintain these speeds even while passing other cars, how many cars will one of the drivers traveling at 120 km/hr pass while traveling one kilometer?

(b) How many passing maneuvers are executed by all drivers in 1 km of road during one hour? (This type of information is directly relevant for accident analysis and prevention.)

Problem 4.3

Show the following:

(a) That if the q vs. k diagram is concave, the v vs. s diagram is also concave.

(b) That if the q vs. k diagram is piecewise linear then v vs. s diagram is piecewise linear.

(c) Devise the *q(k)* relation that would correspond to a shifted exponential *v(s)* curve.

Problem 4.4

A 3-lane freeway drops a lane at location x=0 km. If the maximum flow per lane is 1,800 vph, the jam density per lane is 108 veh/km, and the free flow speed is 100 km/hr, then:

(a) Plot to scale on a single set of axes the q vs. k relations for the 3-lane and 2-lane sections of the road, as well as the curve corresponding to a single lane. Assume that the *Q(k)* curve(s) are smooth and explain in words the geometrical feature that relates the three curves.

(b) Plot the *v(s)* curves that correspond to the above *Q(k)*, and again explain their relationship. (Note here that the spacing of cars in a given lane is not the same as 1/k, and that the s in the *v(s)* relation should be the latter.)

Problem 4.5

Repeat Problem 4.4(a) if the *Q(k)* relation is triangular. Determine the trip time that will hold from x = -1 to x = +2 km as a function of flow when conditions are time-independent, as in *Fundamentals* Figure 4.4d. Recognize that the section under consideration is inhomogeneous and assume that the bottleneck transition is much smaller than the length of the section (3 km). Sketch the answer graphically and express it mathematically.

Problem 4.6

Explain qualitatively what would happen to the solution of Problem 4.5 if the freeway was 3-lanes wide throughout but a point bottleneck (e.g. an incident) that effectively blocked 1 lane existed at x=0 km.

Problem 4.7

A homogenous freeway section of length L is observed to exhibit a concave $Q(k)$ relation under all stationary traffic conditions; i.e. there is a reproducible slope q between the roughly straight $N(t)$ curves at the two detectors for every possible separation, τ. When the observed slope $q_o(\tau)$ and/or the separation between the observed curves $N_0(t)$ is not consistent with that expected, an incident may be suspected. Write an expression for the position of the (point) incident within L in terms of q_o and the observed $N_0(t)$ curves.

Problem 4.8

The purpose of this problem is two-fold: familiarizing you with the organization of the *Highway Capacity Manual (HCM)* and encouraging you to view its recommendations as just that—recommendations.

A two-lane approach to a suburban intersection is located on level ground. There are no left turns or special turning phases. Twenty percent of the vehicles turn right, yielding the right of way to the moderate pedestrian traffic on the cross street. With 10 ½ ft lanes, no buses and only 10% heavy vehicles, parking is allowed because parked vehicles do not encroach on the moving lanes and parking movements are infrequent.

(a) Determine the saturation flow rate according to the 1994 *HCM*, and the minimum proportion of time that the cycle should be effectively green if the approach flow is 1,170 vph.

(b) Critique the procedures for determining the adjustment factors in the HCM formula. (Hint: do you think all the factors in the formula should be independent?)

Problem 4.9

Platoon dispersion (zeroth order approximation): A signal turns green at t=0 and traffic discharges from it at a rate of 3,000 veh/hr for 30 secs until it turns red. If all vehicles reach their cruising speed shortly after the signal threshold (x=0) (free passing) and the distribution of desired vehicular speeds is as follows: 20% at 50 mph and 80% at 60 mph, then draw the cumulative curve of vehicular labels, $N(t)$, at a location x=1 mile downstream of the signal. Solve graphically, and then express the solutions analytically. As a more advanced exercise, express $N(t)$ as a function of x for an arbitrary distribution of desired velocities and the number of passing maneuvers per unit time and unit distance (a measure of accident exposure).

Problem 4.10*

Platoon dispersion (first order approximation): If the fast vehicles of Problem 4.9 are delayed by 5 seconds every time they overtake a slow vehicle (they would follow the slow vehicle for a longer time, of course), but we neglect the infrequent interactions of more than two vehicles due to queuing, then determine the approximate time t_o and location x_o at which all the passing maneuvers will have been completed. Then plot $N(t)$ at locations $x = x_o/2$, $x = x_o$ and $x = 3x_o/2$. (In solving this problem you have developed and used a first order theory of traffic dynamics.) Explain any shortcuts that you use in order to obtain the solution; there are some rather useful ones.

Problem 4.11

Explain qualitatively what part(s) of the answer to Problem 4.10 are independent of the speed of the slow vehicles.

Problem 4.12

Suppose that the $q(t,x)$ and $k(t,x)$ on a one-directional road are given by the following (in an unspecified system of units):

(a) $q(t,x) = q_o e^{(t/t_o - x/x_o)}$ and $k(t,x) = k_o e^{(t/t_o - x/x_o)}$

(b) $q(t,x) = f(x/t)/t$ and $k(t,x) = f(x/t)/x$

(c) $q(t,x) = f(x/t)x$ and $k(t,x) = f(x/t)t$

for $(t,x) \approx (t_0,x_0) > (0,0)$

Identify which of these formulas is physically possible, without traffic generation in the vicinity of (t_o,x_o) and which is not (if any) and explain why. For the physically possible case(s), determine $N(t,x)$ (up to an additive constant) and explain qualitatively what is happening.

Problem 4.13

Relative flow measured by a moving observer in non-stationary traffic: If the conditions of Problem 4.12(b) hold for all $(t,x) > (0,0)$, and we assume that $f(x/t) = 1$, then determine the total number of vehicles seen by an observer that departs location x=1 at t=1 with a speed of v<1, as a function of time and the total seen for $t \to \infty$.

Problem 4.14

Discuss qualitatively how one might go about deriving the trajectory of an interface separating non-stationary states A ($q_A(t,x)$, $k_A(t,x)$) and B ($q_B(t,x)$, $k_B(t,x)$) if we know that the interface passes through a particular (t,x) point (e.g. the origin). As an example, find the interface that passes through point 2,4 and separates $(q_A, k_A) = (1/t, 1/x)$ from $(q_B, k_B) = (1,1)$.

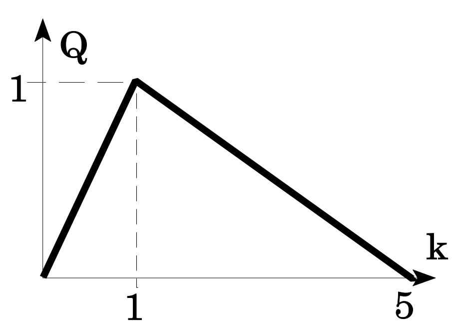
Problem 4.15

Prove that if $Q(k)$ is strictly concave, the relative flow seen by an observer moving with a shock is maximized when the shock is a wave and that the relative flow seen by the wave increases with k. (Show pictorially and prove analytically).

Problem 4.16*

Sketch the trajectory of the vehicle passing through $(t,x) = (0, -1)$ for an initial value problem of a homogenous highway in LWR theory with the following initial data:

$k(0,x) = 1 + x$ (for $-1<x<4$) and $Q(k)$ relation:



Derive its equation and find the times and locations where it crosses the shock and then comes to a stop.

Problem 4.17*

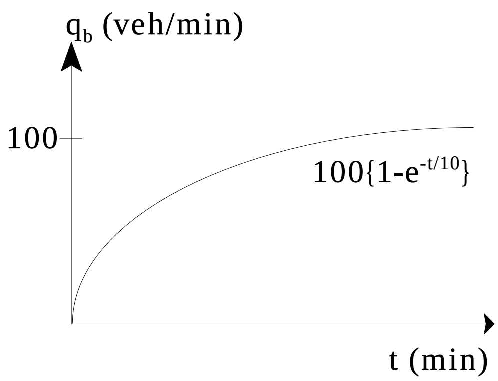
For the data of problem 4.16, derive the equation of the shock path, and then use the minimum principle to write an expression for $N(t,x)$.

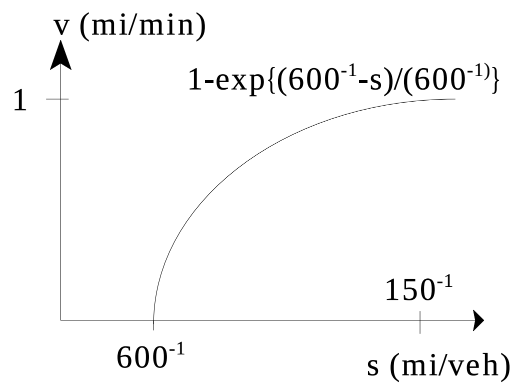
Problem 4.18*

Suppose that the flow past an active bottleneck caused by an incident during the recovery after a total closure is given by a monotonically increasing function $q_b(t)$. If the queue upstream of the bottleneck obeys the LWR model with a concave $Q(k)$ relation, explain how you would construct graphically the curve $N(t,x)$ for a (queued) location that is 1 mile upstream of the bottleneck. Express your procedure precisely.

Problem 4.19*

Solve problem 4.18 for the following data:





Problem 4.20

Consider an imaginary one-directional highway with the following attributes: q_{max}=2,000 vph, k_o=50 vpm and k_j=200 vpm. We assume that a very long queue is discharging past an obstruction at x=0 (miles) with flow q=1,000 vph and that the obstruction is removed at time t=0. If a $q(k)$ relationship (of the two types specified below) holds at all points in time-space, plot the density and flow profiles at x=+½ and x=-½ miles, from t=0 until the flow at both locations is close to q_{max}. Assume:

(a) Two wave-speed relation.

(b) Linear $q(k)$ relation for $k<k_o$ and quadratic for $k>k_o$. The negative wave velocities range from 0 at k_o to -50 at k_j. (These are not necessarily reasonable numbers but will suffice for the purposes of testing your skill).

Make sure that all 4 density profiles are plotted on the same scale. Do the same for the flow profiles.

Problem 4.21*

At the one-mile marker upstream of a point bottleneck with capacity 50 veh/min, traffic flow increases with time as:

$$q(t) = \begin{cases} 100\{1 - e^{(-t/10)}\} \text{ if } t \geq 0 \\ 0 \text{ if } t < 0 \end{cases}$$

where t is in minutes.

If the highway is homogenous upstream of the bottleneck and LWR theory holds with a triangular q - k relation with v_f = 1 mi/min, w = 1/6 mi/min and q_{max} = 100 veh/min, then determine (graphically or analytically):

(a) The cumulative number of vehicles to have joined the queue by time t.

(b) The distance spanned by the queue as a function of t, and the time when the queue spills over the 1-mile marker.

(c) The cumulative number of vehicles to have crossed a location, x_o, ½ mile upstream of the bottleneck by time t.

(d) Check the consistency of answers (b) and (c).

Problem 4.22

Traffic leaving a major event enters a 3-lane freeway at a major on-ramp located 2 miles downstream of the nearest off-ramp. The event traffic reduces the available capacity to the through vehicles as follows:

$\hat{q}_{\max}$ = 100 veh/min for $t < 0$ min
= 60 veh/min for $0 \le t < 10$ min
= 80 veh/min $10 \le t < 20$ min
= 100 veh/min for $t > 20$ min

If the approaching freeway traffic is time independent with q = 100 veh/min and 15% of the traffic exits at the off-ramp (leaving 85 veh/min of through traffic), determine whether or not the freeway queue will back up to the off-ramp. Assume that the q-k relation of Problem 4.20 holds for each lane. Explain what would happen after the spillover. Repeat the exercise assuming that traffic is metered on the on-ramp so that the $\hat{q}_{\max}$ will never dip below 76 veh/min (i.e. so that the $\hat{q}_{\max}$ values would be 100, 76, 76 and 100 if we assume that the same number of vehicles enter). Solve this problem with *(t,x)* diagram and with Newell's method. Check the consistency of both results.

Problem 4.23

A freeway exhibits a triangular flow-density relation with parameters: v_f, q_{max} and k_j, where v_f is the *free-flow speed*, q_{max} is the *capacity* and k_j is the *jam density*.

(a) Plot this curve and derive an expression for the function that gives the (space-mean) speed as a function of density inside a queue. Don't forget to specify the range of k to which the equation applies.

(b) If k_j = 600 veh/mile, v_f = 1 mile/min and q_{max} = 100 veh/min, determine the delay experienced by a vehicle that joins a 2 mile queue caused by a bottleneck that flows at q = 50 veh/min.

Problem 4.24

The cumulative arrival curve at a location 5 miles upstream of a bottleneck *A(t)* is given and so is the departure curve at the bottleneck *D(t)*:

$A(t) = 50t$ for $t < 0$ (min)
$A(t) = 100t$ from $t = 0$ to $t = 30$
$A(t) = 1500 + 50t$ for $t > 30$
$D(t) = -400 + 80t$ for $5 < t < 45$

(a) Draw these curves.

We assume that there is negligible entering/exiting traffic in the 5 mile stretch and that the flow-density relation for the stretch is triangular with q_{max} = 100 veh/min, v_f = 1 mile/min and k_j = 600 veh/mile. If vehicles travel at the free flow speed until they join the queue and from then on they travel at the speed that prevails inside the queue:

(b) Draw the q-k curve and determine the traffic speed (space-mean speed), v_q, inside the queue.

(c) Determine the number of minutes traveled by vehicle number 2000 at speed v_q and the length of the queue (in miles) at the time that vehicle joins it.

(d) Determine the total number of vehicle-hours that are traveled at speed v_q and also at speed v_f by vehicles 0 to 3000.

Problem 4.25

Vehicles traveling on a one-lane road find that it widens to 2 lanes and that shortly after that (1/15th of a mile downstream (352 ft)) there is a traffic signal. The 2-lane section is used to store the queue caused by the traffic signal and in this way increase its capacity. The signal operates with a 60 second cycle and 30 second effective green for this approach. The flow-density diagram for the one-lane section is linear in between the following break-points:

k **(veh/mile)**	***q*** **(vph)**
0.00	0
18.75	900
75.00	1800
150.00	0

(a) Assuming that the diagram for the two-lane section exhibits the same speed for twice the density, draw both diagrams on the same graph. Use only the top half of a sheet of paper.

(b) What would be the maximum flow on the one-lane approach if there was no intersection? What would be the capacity of the intersection if its approach was 2-lanes all the way?

(c) What is the capacity of the intersection now? To solve this part—the main objective of this exercise—you must draw a shockwave diagram on the bottom half of the sheet of paper you used for part (a). For simplicity, you may assume that vehicles accelerate instantaneously to the *optimum speed* after discharging from a queue. You must be careful, however, because the traffic states on the two-lane and one-lane sections of the road come from different q-k curves.

(d) Can you derive a simple formula for the maximum flow as a function of the length of the two-lane section? This should be a two minute exercise based on the part (c) diagram.

Problem 4.26

We consider a homogeneous freeway whose flow-density diagram is linear between the following break-points:

***k* (veh/mile)**	***q* (vph)**
0	0
100	5000
600	0

(a) At exactly 4:00 pm. and when the freeway is flowing steadily at q=4,000 vph, an incident that blocks ½ of the lanes occurs at mile x, ($0 < x < 1$). Determine as a function of x, and according to LWR theory, the exact time at which the drop in flow will be sensed at two detectors located at miles 0 and 1.

(b) Derive an expression for the time t and location x of an incident that is known to have occurred in the road interval (0,1), given the times, U and D, at which a flow drop from Q is q is sensed at the upstream and downstream detectors. (This type of analysis can be used to diagnose detected incidents; detection is more efficiently achieved from occupancy data, however.)

Problem 4.27

Traffic flows at 2,000 veh/hr in the direction of increasing *x* on a one-directional road. At location *x*=0 the road narrows from 2 to 1 lanes. We assume that a flow-density curve, triangular in shape, defines the possible traffic states at all points in time-space. The following parameters apply to the 1 lane section: Free flow speed v_f = 80 km/hr, jam density k_j = 75 veh/km, and optimum density k_o = 25 veh/km.

(a) Plot the flow vs. density curves for the 1 and 2 lane road sections on the same diagram.

(b) Sketch the speed vs. density curve for the 1 lane section. Show clearly where the curve is linear and where it is not.

(c) If a truck located at *x* = 1 km suddenly slows to *v* = 40 km/hr (at *t* = 0), describe in words (one or two sentences) what you think will happen behind it.

(d) Draw a time-space diagram with the relevant interfaces between traffic states and a couple of vehicle trajectories. Find when the effect of the speed reduction is felt at *x* = 0.

(e) When is it felt at *x* = -1 km?

(f) If the truck resumes a speed of 80 km/hr at *x* = 2 km, complete the time space diagram. Include a few more vehicle trajectories.

(g) What is the delay caused by the disturbance to the 100th vehicle behind the truck? (The question has a very easy solution if you see the trick.)

Problem 4.28*

Consider a one-directional arterial with closely but evenly spaced signals and no turning traffic. The signals have been set with identical cycles and phases. The green phases have been coordinated so that a vehicle traveling with the *free-flow speed* would hit the beginning of the green (or the end) at all the signals. This offset is 5 seconds.

Assume that an incident causes a queue to build up past several of these intersections and that queued vehicles behave as in the LWR theory with a constant wave speed that is 5 times smaller than the free-flow speed. The signals have a 30 second green phase and a 60 second cycle. Then,

(a) Sketch the trajectories of the discharging vehicles for the first 4 signals upstream of the incident and determine when the vehicle at the head of this signal's queue would depart. Show all the interfaces.

(b) Sketch the trajectories if the offset is 0 seconds, and the delay to the same vehicles of part (a).

Problem 4.29*

Moving bottleneck with an arbitrary trajectory (Proof of the general statement about *N-spans* at the end of *Fundamentals* §4.4.5).

(a) Sketch some unqueued but irregularly spaced vehicle trajectories to represent free-flowing traffic with vehicle q on a homogenous road. Assume a triangular q - k curve. Then superimpose on this picture a curved segment (with endpoints A and B) representing the trajectory of a slower moving obstruction that interferes with traffic for a finite time. If this (point) obstruction reduces the width of the road by a fixed amount (as in the case of *Fundamentals* Figure 4.24), show on a companion sketch how the trajectories of the vehicles in the first sketch would be modified as a result of the obstructions. Include all the interfaces.

(b) Show from an analysis of the pictures from part (a) that the vehicular trajectories downstream from the x - coordinate of point B do not depend on the path of the obstruction, conditional on the locations of A and B.

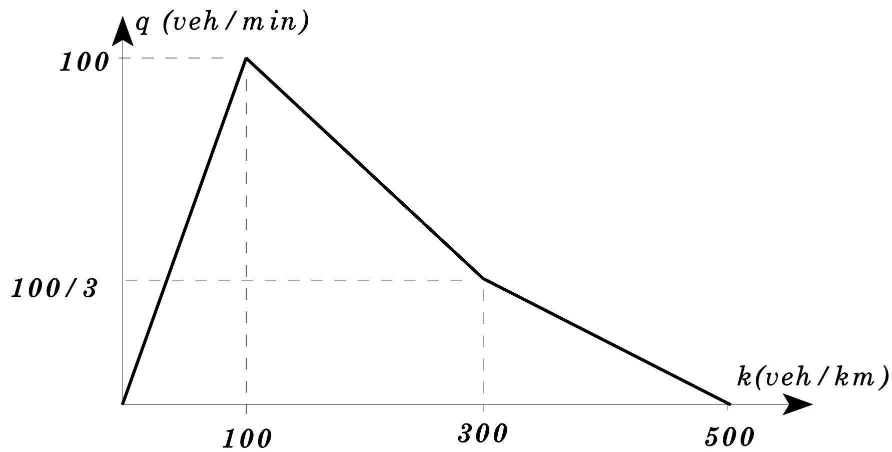
(c) Show that if either point A or B (or both) are moved along the vehicle trajectory on which they lie, the downstream trajectories do not change.

Problem 4.30

Two consecutive intersections that are 100 ft. apart on a one-way street are controlled by identically set, pre-timed traffic signals. Their cycle is one minute and the effective green phase for through traffic is 30 secs. Assuming no turning movements, determine the maximum flow that can get through both intersections as a function of the offset if the spatial extent of queues on the street can be modeled with a triangular *q(k)* relation with the following parameters: jam density = 3 veh every 80 ft, capacity (saturation flow) = 0.5 veh per sec, and free-flow speed = 40 ft per sec. Use 0, 10 and 20 second offsets for illustration.

Problem 4.31*

Suppose that the *q*-*k* diagram on a given highway was as follows:



Construct a *t*-*x* diagram showing the vehicle trajectories and interfaces arising when a capacity stream of vehicles (q=100) is suddenly stopped (e.g. at t=0) at a railroad crossing. (Note that the *q*-*k* curve is not concave).

How long would be the transition seen by a stationary observer situated ½ km upstream of the barrier between the free flow state and the stopped state?

Fundamentals of Transportation and Traffic Operations
Chapter 5 Problems

Problem 5.1

During the rush hour, the desired flows past two neighboring traffic signals in the critical part of a two-way crowded arterial are large enough to generate lasting queues upstream of both signals. The signals are ¼ km apart. They are operated on the same cycle and phases, C=1 min, G=30 secs, and with no offset. The saturation flow of both signals is 50 veh/min.

A secondary street controlled by a yield sign intersects the arterial in between the two signals, x km away from one of them. If the critical gap is H_o=7 secs and the move up time is M=3 secs (both assumed to be the same for everyone), and assuming that right turns have a dedicated lane, find the capacity of the left turn lane as a function of x, and the location x_o where the capacity would be maximum.

You should assume that if a queue backs up past the intersection, the queue will not block cross-traffic but it will block any traffic that would want to join the queue. In order to estimate the extent of queues you should assume that v_f=1 km/min and k_j=300 veh/km.

Problem 5.2*

The entrances and exits of a roundabout are consecutively numbered from 1 to N in the direction of travel. Even numbers are exits and odd numbers are entrances. There is an O-D matrix of (stationary) flows $\mathbf{q} = \{q_{ij}\}$, where $q_{ij} \geq 0$ only if i is odd and j is even.

The roundabout is narrow and each of its links can accept a maximum of 2,000 vph. Likewise, the exits can accept a maximum of 1500 vph. All the entrances are able to carry the respective q_i's.

(a) Introduce the necessary notation to express the capacity condition for $\mathbf{q}$ as a set of linear (in)equality equations.

(b) Assume that q_{ij} = 0, except for $q_{14} = q_{32}$ = 1,500 veh/hr and that there are traffic signals (meters) at entry points 1 and 3. What metering rates μ_1 and μ_3 would maximize the total system outflow?

(c) If the O-D table of part (b) becomes effective at $t = 0$ (min) and lasts until time $t = 30$ (min), and from then on we have $q_{14} = q_{32} = 500$ veh./hr, determine the total vehicle delay that would be experienced if the metering rates are chosen at all times so as to maximize the total system outflow without exceeding capacity. Explain the rationale for such a strategy. (We assume that vehicles traverse the roundabout in a time that is negligible compared with 30 min., our time scale.)

Problem 5.3 *(Courtesy of G. Newell)*

A single airport runway is used for take-offs and landings. The minimum time between a landing and a take-off, a take-off and a landing, or two consecutive take-offs is one minute each, but the minimum time between two consecutive landings is 3/2 minutes. The strategy for sequencing operations is to alternate take-offs and landings whenever there is a queue of both.

If q_T is the rate of take-off requests and q_L the rate of landing requests, determine as a function of the ratio q_T/q_L, the maximum rates q_T and q_L at which one can serve take-offs and landings without having a queue of either steadily increase: i.e., evaluate the *capacity*.

Problem 5.4

Consider a traffic signal at an intersection of two one-way streets with symmetric virtual arrival curves ($V_1(t) = V_2(t)$) and no turns. Using a graphical construction such as that of *Fundamentals* Figure 5.3c as a basis for thinking, derive a formula for the cycle that should be used during the oversaturated period to minimize delay. The cycle should be such that a slight increase in C with the concomitant increase in $\mu(C)$, would induce an increase in the shaded area of the figure that would be exactly offset by the decrease in the unshaded part. Discuss the result for reasonable values of the parameters.

Problem 5.5

A vehicle actuated traffic signal serves two Poisson traffic streams with flows $q_1 = 500$ vph and $q_2 = 300$ vph. The saturation flow rate on both approaches is the same $s =$ 1,200 vph, and the lost time each time the signal changes from one phase to the other is 5 seconds. Then:

(a) Using a spreadsheet, plot on the same graph the cumulative arrival time vs. vehicle number curve for the two approaches. Make sure that the graph includes enough vehicles so that the last vehicle (on both approaches) arrives after 300 seconds.

(b) Using a straight edge and a sharp pencil, construct on such a graph the cumulative departure time curves for all vehicles to have arrived in the first 300 seconds.

(c) Compare the average cycle length you have simulated with the theoretical. Then, describe how you would calculate (from the graphs) the mean and variance of the delay per car for approach 1.

Problem 5.6 *(Courtesy of G. Newell)*

Along a one-way arterial highway of uniform width, there are two one-way cross streets. Vehicles arrive at the intersections on the cross streets at uniform rates $q_2^{(1)}$ and $q_2^{(2)}$, at intersections 1 and 2 respectively, and at intersectoin 1 on the arterial at a uniform rate $q_1^{(1)}$. There is no turning traffic and negligible platoon spreading between the intersections on the arterial. The saturation flow is s at both intersections in both directions, and the effective lost time per cycle is the same at both intersections. Both intersections are controlled by fully actuated two-phase signals which hold any signal phase until the queue it is serving has vanished. If, after the queue vanishes, there is no call for the other phase, the signal phase will continue until there is a call.

Traffic is sufficiently heavy that the signal will typically serve many vehicles in each phase, so one can treat vehicles as a continuous fluid, but the intersections are undersaturated.

Describe how the signals will behave if (a) $q_2^{(1)}>q_2^{(2)}$ and (b) $q_2^{(1)}<q_2^{(2)}$. Would the existence of stochastic effects (Poisson arrivals, for example), slight turning traffic, or slight platoon spreading disrupt the patterns significantly?

Problem 5.7

A traffic signal is set for a 1 minute cycle. The red and green phases on approach A are as follows: R = 30 secs. (including amber phases) and G = 30 secs. If the flow on approach A is constant (no random fluctuations) at 20 veh/min and the saturation flow is 1 veh/sec calculate the average delay for a typical car.

If work on the street is taking place downstream from the intersection so that only 25 veh/min can pass, calculate the average delay caused by the street work to a typical vehicle leaving the intersection. Assume that the queue at the downstream restriction never backs up all the way to the intersection.

If there is a signal downstream of the work zone, with the same (C,G) as the upstream signal, determine:

(a) The delay per vehicle at the downstream signal if the offset between the two signals is the free-flow trip time between them.

(b) The offset that will minimize the total delay.

Problem 5.8*

A railroad classification yard serves a periodic train schedule with a period of 1 week. All the trains are of the *run-through type*. This means that the block of cars for our yard carried by each train is dropped off in one of the receiving tracks immediately after each arrival, and that shortly thereafter (the time lag can be ingnored) the engine (with rest of the train tagging along) collects the blocks of cars that have been prepared for the train in question from a departure track.

A *train* is characterized by the set of blocks of destinations it serves and by its departure time, which is the same every week. We assume for simplicity that each block is carried by only one train. The flow of cars through the yard (by block) is assumed to be reproducible; i.e., to be the same every week.

We also assume that cars can be classified (i.e., moved from the receiving tracks to the clasification tracks) at a given rate during the *classification process*, and that they can also be moved from the clasification tracks to the departure tracks at the same rate during the *train make-up* process.

You are supposed to explain as briefly and elegantly as possible (with pictures and equations as necessary) the following:

(a) An efficient yard operating strategy, which will ensure that no trains are delayed while using as few receiving and departure tracks as possible.

(b) How you would identify the number of receiving and departure tracks needed for your given strategy, assuming:

   (1) That the tracks are distinct, and

   (2) That they are not.

(c) How would you identify the number of cars that would miss a connection, and the time when the yard has the maximum number of tracks in use?

You will be evaluated on the precision of your notation and explanation. Therefore you should be as succinct as possible.

Problem 5.9*

A long freeway with many origins along its way branches off at its very end, when it hits a congested ring road around a central business district (CBD). Branch *1* can serve vehicles at rate α_1 and branch *2* at an unlimited rate. We assume that if a queue develops on the freeway it is never long enough to reach back to any of the origins. (This is not realistic but makes the solution easier). We also assume that vehicles are stored in the freeway and advance into the diverge in a FIFO order, never exceeding the service rate of branch *1*.

(a) Explain how you would determine with pictures and/or equations the total delay in the system if we know the cumulative number of vehicles that leave each origin for each destination as function of time, and the trip time from each origin to the bottleneck.

(b) Formulate the equilibrium conditions for the morning commute problem, where everyone must pass the bottleneck by time t_o, assuming that the number of people traveling from each origin to each destination is known. The goal is to find people's chosen arrival times; i.e. the arrival curves for *1* and *2*. You should assume that people hate time in queue more than the same amount of time wasted by passing the bottleneck too early. You may also assume that the ratio of these two quantities is fixed and the same for everybody. (Hint: explore the possibility of simplifying this problem by studying its equivalence to a simpler one. Describe the analogy in detail.)

Problem 5.10*

A closed-loop corridor of length L consists of a closed-loop freeway with a well defined flow vs. density relation $q(k)$, and the set of arterials within which it is embedded. These can be viewed as a frontage road on which trips average a speed v_a. This speed is both independent of flow and smaller than the free-flow speed of the freeway. We assume for simplicity that the corridor's origin-destination flows are stationary in time and space (along the corridor), that they average D distance units per trip and that the ramp spacing is negligible compared with D. The combined number of trips starting from all origins per unit time is Q. We imagine that we can control the number of trips that enter the freeway per unit time without inducing queues on the ramps, e.g. by pricing, or by a system of stickers whereby people are only allowed to use the freeway on certain days. The remaining trips use the arterials, resulting in a combined arterial flow q_a across every cross-section of the corridor.

(a) Write a relation among the freeway flow q, the arterial flow q_a and the regional origin-destination flow Q. (Hint: it will involve L and D.)

Note that q_a can be freely chosen and that q is then determined by the relation you derived in (a). Alternatively, you could imagine choosing the density k_a and determining k (or q).

(b) In view of this, and using the relation $q(k)$, rewrite the result of part (a) in terms of Q and densities k and k_a.

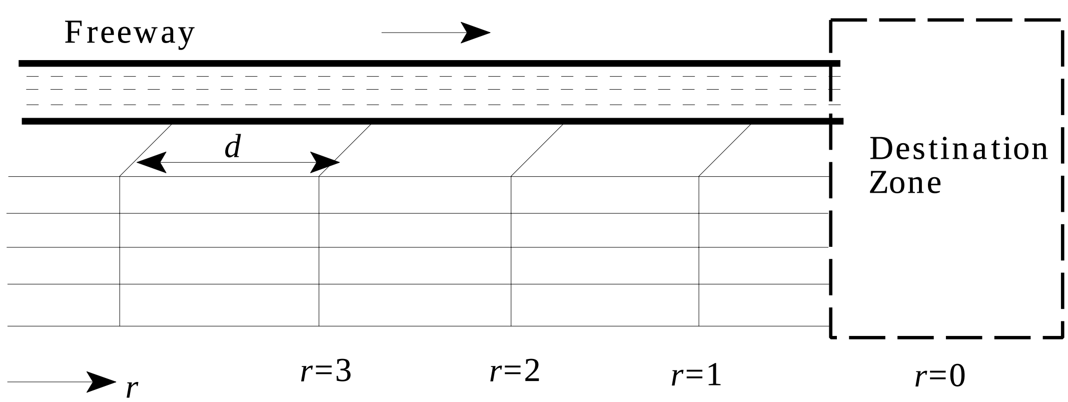
(c) Find the (q,k) combination that minimizes the *total system delay* and then describe a simple graphical construction to display it on the $q(k)$ diagram.

(d) Repeat (c) under the assumption that there is no control and users reach an equilibrium where the speeds of both facilities are the same. Calculate the increase in delay from the solution of part (c), and express the result on the $q(k)$ diagram.

Problem 5.11

Imagine a network consisting of three components: a freeway corridor, a set of arterials and an inexhaustible traffic demand for a "destination zone," *D*. Drivers are assumed to choose to enter the freeway only if its travel time (including any queueing on the on-ramp) is less than the arterial trip time. The freeway is homogeneous with *m* lanes, so that the average speed within a queue is a function *V(Q)* of the discharge rate, *Q*, for any $Q \le q_{max}$. The freeway has some freeway flow speed, v_1, and the arterials have an average speed (including stops), v_2. The freeway on ramps are spaced some distance *d* apart. We assume that when the freeway and the on-ramp are queued, merging vehicles and vehicles in the right-most freeway lane combine into one traffic stream in a one-to-one rate.

Determine the equilibrium queues on each ramp as a function of *r* if people choose the least time route. Would a ramp metering strategy change the rate at which vehicles enter *D*? What would be the advantages and disadvantages of metering/closing the ramp closest to *D*?



Problem 5.12

Repeat Problem 5.11 if the freeway is the only way to reach *D;* i.e. if the arterials end before reaching the destination. Would a ramp metering strategy change the rate at which vehicles enter *D*?

Problem 5.13

A capacity "paradox:" There are two ways of going from *A* to *B*. A fast route that can be accessed after passing through a bottleneck of capacity μ (veh/min) and a slow route that takes τ (min) longer. The bottleneck queue can be seen from the point of decision, so that drivers can estimate the delay due to the queue, w. We also assume that if the queue contains more than Q_{max} vehicles it will have spilled over past the diverge, and that at the diverge drivers choose the fastest route (e.g. the fast route if $w<\tau$). Then, if the demand during the rush hour (λ, assumed to be constant during the rush) exceeds both μ and Q_{max}/τ, determine the rate at which vehicles will advance into the diverge during the rush if (a) $\mu < Q_{max}/\tau < \lambda$ and (b) if the bottleneck capacity is expanded: $Q_{max}/\tau < \mu < \lambda$.

Fundamentals of Transportation and Traffic Operations
Chapter 6 Problems

Problem 6.1 *(Source: Anonymous)*

A seven-story building is served by two elevators. You are at the ground level, trying to decide whether to use the stairs or wait for an elevator. The elevators operate independently, and each is equally likely to be at any floor at any given moment.

(a) Sketch the sample space for the position of both elevators. Floors are numbered from 1 (ground) to 7, and neither elevator is at the ground floor (they could be at floors 2, 3, 4, 5, 6, or 7).

(b) Use the sketch to determine the probability that at least one of the elevators is no more than two floors away from the ground level.

(c) Use a new sketch and repeat the above exercise assuming now that you are on the third floor, and neither elevator is there (they could be at floors 1, 2, 4, 5, 6, or 7).

Problem 6.2

If X, Y are the coordinates of a randomly selected house in a circular city of radius 1, $0 \leq X^2 + Y^2 \leq 1$, where housing density is uniform:

(a) Are X and Y statistically independent? Identically distributed?

(b) What is the probability density of a house's euclidean distance to the CBD?

(c) What is the expectation?

If the travel must take place over an infinitely dense square grid network oriented with the coordinate axes,

(d) What is the expected distance traveled to the CBD?

(e) What is the probability density of the distance traveled to the CBD?

Problem 6.3

The distance between points *A* and *B* of a certain highway is measured as the sum of 124 independent measurements. The random error *E* in each of the measurements is uniformly distributed between [-2, 2] cm. If the total distance *AB* is approximately 2 km and the lengths of the segments are approximately equal, compute the following:

(a) The mean and variance of the total error in the distance *AB*.

(b) The probability that the total error is not more than 0.01% of the actual distance *AB*.

Problem 6.4

Suppose 4,000 containers are to be loaded onto a vessel. Each container is weighed before being loaded using a heavy-duty scale that indicates their weight accurately as an integer number of pounds (so that the (round-off) error in each measurement is uniformly distributed on [-0.5, 0.5]).

(a) What is the maximum possible error in the total estimated weight?

(b) Would it be reasonable to use a normal approximation to estimate the error in the total weight measured? Explain.

(c) What is the probability that the total weight will be underestimated by 50 lbs or more?

Problem 6.5

A container crane unloads 1,000 containers from a ship. All of these containers have an equal probability (1/4) of being 20-footers. The rest are 40-footers. Twenty footers have a 0.4 probability of leaving by train, while 40-footers have a 0.6 probability.

(a) What is the mean and standard deviation of the number of 20-foot containers leaving by train?

(b) What is the mean, standard deviation and approximate distribution of the number of TEU's (twenty-foot equivalent units) unloaded? Hint: This question is very easy but a little tricky because the numbers of containers unloaded of each type are not independent; their sum is 1000. (Note: one 2-foot container = 1 TEU and one 40-foot container = 2 TEU's.)

Problem 6.6

A train with 88 railcars is being classified at the railyard. All of these cars have an equal probability of being sorted onto any of 10 available classification tracks. What is the mean and standard deviation of the number of cars sorted to track 1?

Each of those cars is carried by the next departing train with a probability $p = 0.3$. What is the mean and standard deviation of the number of cars remaining in track 1?

Problem 6.7*

Consider the uphill direction of a very long (infinite for our purposes) two-lane, bidirectional road that climbs to a mountain pass. Geometric design considerations preclude passing, except at wider passing sections, of length l (miles), which are regularly spaced (beginning to beginning) L miles apart. The flow of trucks in the uphill direction is q and the flow of cars: $Q = nq$, where n is assumed to be an integer.

Trucks travel at speed v and cars travel at speed V ($v < V$) unless trapped in a moving queue behind a truck. The first car in a queue is assumed to initiate a passing maneuver by accelerating instantaneously to speed V when the car (not the truck) reaches the passing section's threshold. Passing is only possible inside the passing sections. Able to accelerate and decelerate instantaneously between speeds v and V, cars are also assumed to keep a headway of 2 seconds at all times with the car they follow. (This means that if a car suddenly changes speed, a trailing car will change speed at the same point in space.) Then:

(a) If $l = 0.25$, $L = 5$, $q = 2$ veh/min, n=5, $v = 45$ mph and $V = 60$ mph, find the average speed of a fast vehicle as it negotiates the uphill section. (Assume that on approaching the uphill section the trucks are evenly spaced and that each is followed by the same number of cars.)

(b) As a function of q, n, v and V, determine the smallest L and l that will ensure that long term average speed of the cars is 60 mph (no delay). Do first for the data in (a).

(c) Explain why the number of vehicles trapped behind a truck on approach to the uphill may vary across trucks like a Poisson random variable. Then, give reasons why it might not.

(d) If the number in (c) is Poisson, explain how this would change the answer to part (b).

(e) If, for the data of part (a) with $n=0$, the headway between trucks are described by a Poisson process with rate q, determine the average travel speed of an isolated car. (Hints: A car will always leave a passing section at speed V; and the time until it meets the next truck should have the forgetfulness property.)

Problem 6.8*

The intersection of two one-lane, one-way streets is controlled by a stop sign. Traffic flow on the major street is well described by a stationary Poisson process with mean Q (veh/sec). Queued vehicles in the minor street start looking for gaps M seconds after the preceding vehicle in the queue has merged. They merge (immediately) upon finding a gap in traffic greater than T secs (T > M). Find the minor street capacity as a function of Q, T and M. (Hint: Find first the expected number of merges in one main street headway.)

Problem 6.9

The headways of successive cars observed at an active bottleneck are uniformly and independently distributed between a and A secs ($a < A$). Use the inversion formulas for processes with positive and independent increments to derive the index of dispersion of the counting process when the latter is observed on scales including many vehicles.

Problem 6.10

Use a spreadsheet to do the following:

(a) Generate 500 sets of three random digits (rectangular (0,1) variables), (X, Y, Z) and their average $S = (X+Y+Z)/3$.

(b) Calculate the theoretical mean and variance of S. Compare with the sample mean and variance for the 500 observations of S.

(c) Use the *SORT* capabilities of the spreadsheet to plot the empirical c.d.f. of S, and the theoretical one according to the central limit theorem (this may require that you integrate numerically with your spreadsheet the normal p.d.f.). Comment.

Problem 6.11

Export containers arrive at a port marshaling yard, where they accumulate while waiting for the departing ships. Type a containers are 20 feet long and take 200 ft^2 of yard floor space; type b containers are 40 feet long and take 400 ft^2 of yard floor space. Both container types arrive at the yard as independent Poisson processes; each with an average arrival rate of 60 containers per day—this arrival pattern holds around the clock, 7 days a week. Containers of each type enter the port facilities through separate gates, where they are inspected (one at a time) for a nearly constant 5 minute period.

Answer the following questions:

(a) What is the probability that the next container to arrive at one of the gates immediately after a container that doesn't have to queue can also avoid queuing delay? Explain the logic behind your answer.

(b) What is the probability that more than 75 type b containers arrive in 1 day? Be explicit about any approximations you use.

(c) The port is considering including a more formal inspection at the gate (including random customs searches), taking 20 minutes on average, with a standard deviation of also 20 minutes. Approximately, what would be the steady state average queue length at the gate with the expanded inspections?

(d) Write an expression for the mean and the variance of the total floor space taken by the containers of both types together arriving in a time interval of length τ days. Give the answer for $\tau = 7$ days.

(e) Assume that the arrival rate of type a containers is not known exactly (it is known to be between 45 and 55 containers per day) but that in your zeal for accuracy you want to pinpoint it within a standard error of 0.1 containers per day. How long would you have to observe the system?

Problem 6.12

Let $\{S_1, S_2, ..., S_n\}$ be a number of independent vehicle speed measurements from an unknown distribution with mean μ and variance σ^2. Determine the smallest sample size *n* such that the probability of the absolute difference between the true mean μ and the sample mean S being greater than one tenth of the true standard deviation is less than 0.05:

$$P\ (|S - \mu| > \sigma/10) < 0.05.$$

(You may assume that *n* is large.)

Problem 6.13

Comparison of *N*-curves: The true (unknown) one-minute counts, *n(t)*, upstream of a bottleneck depend on the time in the middle of the counting interval, *t*, as follows:

$$n(t) = n_o(10 + \sin t),$$

where *t* is in minutes. (A simple expression is used here for simplicity of formulation, and so that the various error measures can be computed quickly; the qualitative results arising in this problem do not depend on the ability to express the data in a neat closed form).

Model *A* predicts the following interval flows (i.e. minute counts): $n_A(t) = 10n_o$. Model *B* predicts: $n_B(t) = n_o(9.8 + \sin t)$.

(a) Plot the curves $n(t)$, $n_A(t)$ and $n_B(t)$ on the same graph and evaluate the root mean square error in the counts produced by model *A* and model *B* for a long (1 hour) period.

(b) Plot the cumulative curves obtained for 1 hour of data $N(t)$, $N_A(t)$ and $N_B(t)$ and again determine the root mean square error in the counts produced by both models. On the basis of the results in (a) and (b), which model would be preferable?

Problem 6.14

In the case of Problem 6.13, the model predictions remain fixed, but the true counts fluctuate from day to day. On day i they are $n(t)_i = n_o\,(10 + \sin t) + \epsilon_{it}$ where the ϵ_{it} 's are i.i.d. observations from a random variable with mean 0 and variance $(2n_o)^2$. Then, identify the time t_o at which the maximum deviation between $N_i(t)$ and $N_A(t)$ would be expected to occur, as well as its distribution. If the model is rejected if its maximum deviation (which we approximate by the deviation at time t_0) exceeds the 95th percentile deviation that would occur at t = 60 with a true model, determine the probability of rejection on any given day. Repeat for Model B.

Problem 6.15

Traffic on a short stretch of road (length, L=1 km) which is being studied by time lapsed photography can be described as a superposition of N (stationary) Poisson streams (i=1, ...N) with speeds v_i and average flow q_i. If the time lapse Δt is short or comparable with a vehicle's trip time:

(a) Show that the number of vehicles, n_m, on each photograph, m, is an outcome of a Poisson random variable with a given mean. Find the mean.

(b) Find an expression for $c(l\Delta t)$, the covariance in the counts of two frames, l shots apart. Plot a graph of $c(l\Delta t)$ vs. $(l\Delta t)$ if N=2, q_1=10 veh/min, q_2=20 veh/min V_1=2 km/min and V_2=1.5 km/min. Then evaluate the covariance integral, C, and the variance of $\bar{n}$ as a function of time if observation lasts many minutes and $\Delta t \to 0$.

Problem 6.16

The table below contains vehicle positions and speeds, as observed from two aerial photographs of a 1 km freeway section.

	DATA	
Vehicle #	**Position (m)**	**Speed(m/sec)**
1	27.14	23.28
2	121.34	24.94
3	143.16	22.77
4	168.64	25.84
5	204.41	26.87
6	253.93	24.93
7	291.11	26.26
8	382.08	25.07
9	519.82	25.74
10	529.76	22.31
11	531.51	22.21
12	570.41	22.97
13	654.68	22.69
14	665.61	25.42
15	671.45	24.46
16	689.56	25.84
17	707.62	26.41
18	773.21	26.12
19	824.86	26.42
20	839.49	26.45
21	915.17	26.56

Using these data:

(a) Estimate the time mean speed.

(b) Construct the empirical cumulative distribution function of the speeds that would be seen by a stationary observer. (Assume that the observed pattern is repeated every km of freeway, and that the vehicle trajectories are straight). Calculate the mean and variance.

(c) Compare the result of (a) and (b) and try to explain any discrepancies.

Problem 6.17 *(Courtesy of G. Newell)*

Suppose that an observer knows that her vehicle detector turns on and off before the vehicle has completely crossed the detector. From experimental calibration of the instrument she has verified that the true length of a vehicle l_j is always $\in$ larger than the effective length for all vehicles (independent of j). She knows $\in$. She observes a flow q and an effective occupancy r' from her detector. What else must she know in order to convert her r' into r so that she can compare her observations with someone else's? (The vehicles have different l_j and different speeds v_j.)

Problem 6.18

Paired loop detectors can be used to estimate vehicle length and speed. If the length of each detector is d and their center to center separation is $D >> d$, derive an expression for the speed and length of vehicle i (v_i, l_i) as a function of the times t_1^u and t_2^u when the upstream detector u goes *on* and *off*, and the times t_1^d and t_2^d when the downstream detector does the same. You may assume in your derivation:

(a) That $l_i < D\text{-}d$ for all i.

(b) That the detectors are *on* when any part of the vehicle is over the detector.

(c) That vehicles maintain their speed over the detectors.

Explain how a bias in the synchronization of the 2 detectors would affect the results.

Problem 6.19*

Explain how you would determine the acceleration of vehicle i with the setup of Problem 6.18, assuming that vehicles undergo a constant acceleration motion over the detectors. Ignore detection errors.

Problem 6.20

Consider the numerical data of Problem 6.15 (b) and assume that a moving observer traveling with the traffic stream at speed v_o is used to estimate the (stationary) and unknown flow ($q = q_1+q_2 = 30$ veh/min) on the 1 km road section of interest. She does this by recording in a single one-way trip the relative speed of every vehicle she passes or leaves behind; i.e. the number of vehicles of type 1 and type 2, m_1 and m_2. Then,

(a) Write a formula for an estimator of q in terms of m_1, m_2, V_1, V_2 and v_o. (The observer is aware that there are only 2 vehicle families)

(b) Derive an expression for the variance of estimator as a function of q_1, q_2 and v_o and plot it as a function of the amount of time spent in the trip ($1/v_o$).

Problem 6.21

This problem examines the effectiveness of license plate travel time studies. An arterial street carries a flow of q veh/hr for its whole length. However, each individual vehicle has a probability $p = 0.2$ of leaving the arterial at each intersection; i.e. at every intersection 20% of the traffic turns off (and an equivalent number of vehicles turns onto the arterial). The method used involves recording a vehicle's license plate and time of arrival at the two ends of the study section by two separate observers. The study section contains N intersections. Each observer can record a maximum of μ cars per hour. The (unknown) average travel time to travel one block is $E(t_o)$. Its variance (across vehicles) is $\delta E(t_o)$, where δ is a time constant. We assume that these moments are (approximately) the same for all blocks. If the travel times on succeeding blocks by the same vehicle are mutually independent, then the variance to mean ratio of the time to travel the study section is $N\delta E[t_o]/NE[t_o] = \delta$. Under these assumptions, answer the following questions:

(a) For light traffic ($q < \mu$), write an expression for the duration, T, of the study period that will allow us to estimate the average travel time for the section with a standard error equal to 5% of the mean. The expression should include as variables: q, n, δ, and $E[t_o]$. Explain why T is smallest for $N \approx 4$ or 5, and what you would do if you had to study a very long arterial with, say, $N = 30$. What is the optimal T if $q = 400$ vph, $N = 4$, and we believe that $E[t_o] = 30$secs. and $\delta = 20$secs.

(b) Derive an expression for heavy traffic ($q > \mu$).

Problem 6.22

A few bus lines ($i = 1....N$) connect at a given terminal. The station is so small that walking times between connecting vehicles and or the entrance/exit of the terminal are comparable to the bus stopping time and much smaller than the headways. Observers record the cumulative arrival and departure curves of passengers vs. time at each berth, but not at the entrance and exit of the terminal (i = 0). Explain how you would determine from this information (and from the realized schedule of bus arrivals and departures) the cumulative flows by O-D pair including the terminal itself (i = 0) as an origin and destination. You may assume that the data are consistent with a world in which passengers do not waste time moving around.

Fundamentals of Transportation and Traffic Operations
Chapter 7 Problems

Problem 7.1

Draw a time-space diagram including the trajectories of 2 rapid transit trains that undergo the same motions while separated at all times by a 1 minute headway. The two trains pass through and stop at two transit stations that are 1 mile apart. They stop for ½ minute at each station and travel at 60 mph in between. Label the headway, show the position of the stations, identify on the figure the time (t^*) when the two vehicles are closest in space, and determine such minimum distance.

Problem 7.2

(a) If arrivals at a bus stop are stationary in time (constant flow) and the headways alternate between 5 minutes and 10 minutes, determine the probability that a randomly chosen passenger has to wait more than x minutes $G_w(x)$, for all possible x. Plot the result. Would the result change if passengers arrived independently of the schedule as a stationary Poisson process?

(b) If the same number of passengers arrive to meet each bus and they are uniformly distributed in a 7-minute window of time that extends all the way to each bus' departure, determine $G_w(x)$ and the average waiting time assuming that people will board the first bus that comes.

Problem 7.3 *(A similar, simpler version of this problem can be found in Horonjeff & McKelvey, "Planning and Design of Airports," 1983)*

Two different aircraft use a runway used for landings only. One aircraft has an airspeed of 120 knots, while the other has an airspeed of 150 knots. The common approach path is 6 nautical miles long. Along this approach, the aircraft must maintain a minimum separation distance of 3 nautical miles. Both aircraft occupy the runway for 60 seconds, and they cannot be on it simultaneously. There is a 20 knot headwind.

(a) Determine the minimum safe aircraft headway at the runway threshold for each pair of airplanes.

(b) If the arriving aircraft are split 50-50 between those with airspeeds of 120 and 150 knots, determine the maximum theoretical (no buffers) runway capacity in aircraft per hour.

Problem 7.4 *(Courtesy of G. Newell)*

Each of two buses carries passengers from a depot to various destinations and returns for another trip with a round trip time very nearly equal to R. The buses are run by independent drivers, however, who make no attempt to coordinate their schedules. Actually, one bus runs slightly faster than the other so that over many trips the fraction of trips that the second bus leaves within a time t after the first bus is t/R, with $0 < t < R$. In effect, the times between departures of the buses are random, with a uniform distribution over the interval $0 < t < R$.

If the passengers arrive at the depot at a constant rate, what is the average time that a passenger must wait for the next bus? Compare this with the wait if the headways are controlled so as to be $R/2$.

Problem 7.5

A bus takes commuters from a CBD to a suburb on a route that (not including stops) takes on average ½ hour; we also know that the non-stop travel time is normally distributed with a 2 minute standard deviation. The bus makes 20 stops, and 50 passengers start the trip. Each stop takes a time which is the sum of a constant (30 seconds), plus a negative exponential random variable with mean 10 seconds. In addition, each alighting passenger extends the time of its stop by a time which is rectangularly distributed between 2 and 12 seconds. How late after the start of the trip can the bus company be 95% sure that the bus will be available for other purposes?

Problem 7.6 *(Courtesy of G. Newell)*

Suppose that two shuttle buses operate between downtown Berkeley and the campus. One bus can carry 20 people and has a round trip time of eight minutes (including loading and unloading passengers), the other can carry 25 people but has a round trip time of 10 minutes. There is always a queue of people waiting at the downtown bus stop, but people arrive at the campus bus stop at a steady rate of 2 per minute.

If neither bus is idle at any time, determine:

(a) Flow of people from downtown to campus.

(b) Flow of people from campus to downtown.

(c) Flow of buses from downtown to campus.

(d) Flow of buses from campus to downtown.

(e) The average number of passengers per bus from campus to downtown.

If one insisted on dispatching these buses from downtown to campus at equal headways, what is the maximum number of people that could be carried per hour from downtown to campus? Would you expect this strategy to reduce the average waiting time of passengers going from campus to downtown?

Problem 7.7

Consider a symmetric circular bus route, where every passenger travels exactly on 4 links (he alights on the 4th stop not counting the one he got on) and where the cumulative flow of arriving passengers at each stop is $A(t)$. If the headway is H and the trip time between stops (not including the stop) is $H/2$, depict graphically and express analytically the occupancy of the bus immediately after departing a stop at time t^*. Repeat the problem if, instead of 4 stops, there is a distribution where p_i of the passengers travel for i stops (i = 1, 2, 3, ...).

Problem 7.8

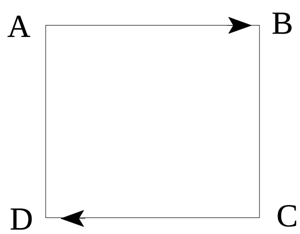
A 200 double-chair lift at a ski resort (a form of transportation) moves at 8 mph. Such a chair lift can hold at most 200 skiers, since at any point in time only 100 two-person chairs are going uphill. The distance traveled by each chair from the bottom to the top is ½ mile. The downhill run is 1 mile long. On Sunday they expect 500 skiers. Then:

(a) If everybody skis downhill at 16 mph, find the length of the lift line that will develop (i.e. the number of skiers waiting for a chair), and the average time it takes to go through the line.

(Note: to solve this problem you should recognize that: (1) the flow of skiers going up must equal the flow of skiers going down, and (2) that if a line develops, the chair must be full to capacity.)

(b) If half the skiers ski back downhill at 24 mph and the other half at 10 mph, then determine the length of the lift line, and the number of 24 mph skiers it will contain.

Problem 7.9



Transportation service is to be provided by M=4 buses linking stops A, B, C, and D, as shown by the route in the figure. The 4 links on this route have the same length so that, aside from the passenger loading and unloading time a bus will take τ=300 seconds to traverse the link between any two consecutive stops. This includes the time needed to stop the bus and open and close the doors. The passenger flow between any two different stops (e.g. A and B, D and B, etc...) is q=60 passengers per hour per origin-destination pair; each boarding and alighting passenger extends the link traversal time by δ=5 seconds.

(a) *Numerical Analysis:* Answer the following questions assuming that the buses can maintain constant headways operating continuously (without slack time), and that passengers for any O-D pair arrive at a constant rate over time.

(1) What is the flow on each link?
(2) What is the headway at each stop?
(3) What is the minimum bus capacity that can do the job?
(4) What is the passenger delay before boarding the bus, averaged over all passengers?
(5) What is the total average time in the system (delay plus riding time)?

(b) *Analytical Modeling:*

(1) Derive an expression for the total average time in the system for an arbitrary number of stops, n, and arbitrary: q, τ, δ and M.

(2) The bus company should decide whether to assign $M/2$ buses for clockwise service and $M/2$ buses for counter-clockwise service. In this manner, a passenger traveling from A to D could take the counter-clockwise bus and cut her travel time substantially; on the other hand this might increase the headways and the average waiting time as a result. Which of these two strategies yields the least total average time in the system depends on the variables (q, n, τ, δ and M). Can you state a condition for choosing? Which of the variables are irrelevant?

Problem 7.10

An entrepreneur is planning to open a long term parking lot d kilometers away from an airport. To be competitive, he plans to operate shuttle buses between the lot and the airport at regular H minute headway, 24 hours a day. He plans to use buses able to carry any number of passengers. The buses would travel at speed v (km/min) and would spend T_1 and T_2 minutes stopped at the two ends of their route (the lot and the airport).

Outbound customers (traveling from the lot to the airport) are foreseen to arrive at an evenly distributed rate of q cars per minute throughout the day, and the inbound (return) passengers to behave similarly. We also believe that the probability density function of the parking times, P, would be:

$$f_P(x) = (1/p)\, e^{-x/p}, \text{ if } x \geq 0, \text{ and } f_P(x) = 0, \text{ otherwise.}$$

Finally, we assume that the operating cost of one bus (driver, wages, lease cost, etc.) is w (\$/min), that the rent for one parking space is r (\$/car-min), and that all other costs are negligible. Then,

(a) Derive a formula in terms of some of the above variables for the rate at which the entrepreneur's cost per car served increases with d.

(b) Calculate the required size of the lot, assuming that its surface area should be double of that which is strictly needed to hold the average number of cars in the lot.

(c) Write an expression for a parking fee schedule as a function of the actual parking durations (p_i) experienced by specific individuals (i) if the goal is to achieve a 50% mark-up over the cost of providing the service.

Problem 7.11

Explain technically how you would go about calculating the total emissions of NO_x in lbs. per hour arising from a 1 mile long freeway segment in which traffic travels at speed v with flow q, assuming that you are given curves such as those in the *Fundamentals of Traffic Engineering* (Homburger, *et al.*, 1996) that relate NO_x emissions by a single vehicle (in lbs per vehicle-mile) to its speed.

Problem 7.12*

Because of tides, an inland port's shallow river access can only be entered during two 3-hour periods each day. This causes a delay to ships waiting to enter the port; and also to those wishing to leave. The port authority is contemplating a dredging project that would essentially eliminate all the delays, at a substantial cost. Assuming that the opportunity cost of ship delay is $ 25,000 per hour waited and that the port expects 200 ships each year, your job as a transportation specialist is to determine the economic benefit from the dredging project. You are to:

(a) Develop a yearly benefit estimate if ships arrive randomly in time, travel the river section in one hour (the one way access time is T_A=1 hour) and spend T_P=17 hours in port.

(b) Describe how the problem under consideration is analogous to a traffic flow problem through an arterial street section controlled by two traffic lights. (Include a brief description of both systems, explain any assumptions needed for the equivalence to hold, and identify the parameters/variables that are mathematically equivalent).

(c) Develop an expression for the annual savings as a function of the ship turnaround time, $T = T_A+T_P$, and plot it.

(d) Without any complex calculations (an argument worth two or three lines of text suffices), plot the curve of part (c) if the ship service times are random with a standard deviation that is large compared with 12 hrs.

(e) Explain how your results might change if ships can be scheduled to arrive any time (e.g. when the tide is up).

Problem 7.13*

Cities *A* and *B* are connected by a congested bridge (*1*) on which there is a steady state flow of *Q* cars per hour. There is one person per car and no toll is charged. You believe that an increasing relationship of the form $y = c(x)$ exists for the time delay to the users of the bridge *y* (in hours) when the flow is *x* veh/hr. Because *y* is large, we contemplate the idea of allowing an entrepreneur to build and operate a parallel identical bridge (*2*) with the same delay function, *c*; and to charge any toll, τ ($).

We agree with the entrepreneur that the flow from *A* to *B*, *Q* , would not change and that the flows on the two bridges, $x_1 + x_2 = Q$, would be the result of an equilibrium in which each driver would choose the cheapest bridge, $i = 1, 2$, as measured by the two generalized costs, Z_i ($), experienced by the driver.

These costs Z_i are defined for each bridge as the sum of the toll, $\tau_i = 0$ or τ, and the cost-equivalent of delay at equilibrium, αy_i, where α ($/hrs), is a characteristic of each driver that varies across drivers. The fraction of the population with $\alpha \leq z$ is given by a known function, *F(z)*.

(a) Set up the equations for the toll that maximizes the entrepreneur's revenue. (This is what the entrepreneur would do if the cost of operating the new bridge is independent of flow).

(b) Find the toll if $Q = 10{,}000$, $c(x) = 1 + (x/5{,}000)^4$, and $F(z) = z/10$ for $0 \leq z \leq 10$.

(c) Calculate the increase in consumers' surplus ($/hour) induced by the new bridge (1) for the users of the new bridge and (2) for the remaining ones. (Aside from an irrelevant additive constant, surplus is the negative sum of the generalized costs across drivers).

(d) Repeat (c) if we choose to express consumers' surplus as the sum of the generalized delays experienced, defined for each driver as: Z_i/α. Comment on the results of (c) and (d).