Using the Input-Output Diagram to Determine the Spatial and Temporal Extents of a Queue Upstream of a Bottleneck

By

Tim W. Lawson
David J. Lovell
and
Carlos F. Daganzo

Institute of Transportation Studies
University of California, Berkeley
Berkeley, CA 94720

March 1997

Abstract

This paper describes a simple approach for modifying an input-output (or queueing) diagram to measure the time and distance spent by vehicles in a queue in a much simpler and self-serving manner than using a time-space diagram. The graphical technique requires construction of a curve depicting the cumulative number of vehicles to have reached the back of the queue as a function of time, but as shown herein, the technique can be easily automated with a spreadsheet. Application of the technique is shown for the simple case of a constant departure rate from a bottleneck, and for the slightly more general case of a bottleneck capacity which changes once, which is demonstrated to be applicable to the study of an undersaturated traffic signal. In the course of describing the usefulness of this technique for estimating several measures, including the maximum length of a physical queue and the time at which this maximum occurs, the paper clarifies the difference between "delay" at a bottleneck and the "time spent in queue," which appear to have been confused in some of the literature.

INTRODUCTION

This paper describes a simple approach by which an input-output (or queueing) diagram is modified to show the time and distance individual (and aggregated) vehicles spend in queue upstream of a bottleneck. The technique involves the construction of a curve showing the time that each vehicle reached the back of the queue; or its analogue, the cumulative number of vehicles to have reached the back of the queue by time *t*. This construction permits many measures to be read directly from the input-output diagram, in a much simpler way than using the more conventional time-space diagram, which can be quite laborious to construct. While not the first example of these different diagrams being used in a consistent manner (see, e.g., (*1*,*2*)), this paper demonstrates the usefulness of our approach, and addresses the confusion in some of the literature surrounding the distinct concepts of "delay" and "time in queue."

Delay represents the difference between the time a vehicle actually took to traverse a given distance and the time it would have taken if it were unobstructed. Delay is an appropriate measure to use when studying the impacts of congestion on people's time. When evaluating instead the energy and emissions implications of alternatives, for example, the more appropriate quantity to evaluate is the amount of time actually spent in queue (*3*) ("waiting time" or "time in queue"), which is greater than the delay. This relationship should be intuitive, because vehicles traveling at free-flow speed would naturally reach the location of the back of the queue (which has physical length) before they would have reached the bottleneck without obstruction. The paper will show that, under certain circumstances, the time a vehicle actually spends in queue is a constant multiple of its delay, and that this constant is independent of the arrival pattern of vehicles to the bottleneck.

This paper begins by examining the case of a bottleneck with constant capacity. The more conventional (time-space diagram) approach is described before the proposed graphical approach is demonstrated, in order to show the consistency of the two techniques. Numerical expressions applicable to the proposed approach are developed, along with procedures to automate calculations on a spreadsheet. The paper then extends the technique to the slightly more general case of a bottleneck capacity which changes once at a known time. The graphical approach is described, as are the corresponding numerical expressions, along with a detailed interpretation of the results. The technique is next demonstrated with a very familiar scenario, the undersaturated traffic signal. Finally, the benefits and limitations of the technique are discussed in the conclusions.

CONSTANT DEPARTURE RATE

We first consider the simplest case of a bottleneck with a constant maximum departure rate m, using both a more conventional (time-space diagram) approach, and the proposed (input-output diagram) approach. In both instances, we assume that a constant free-flow speed v_f holds for all uncongested traffic (independent of flow), and that whenever congestion occurs upstream of the bottleneck, vehicles traverse the queue at some constant speed v_m (dependent on the bottleneck flow), which is less than the free-flow speed. We also assume, for sake of simplicity, that speed changes occur instantaneously.

(Any real difference from this is of little significance and would detract from the discussion.) Finally, we assume that vehicles neither enter nor leave the traffic stream while in the queue (which averts the need for a much more complex treatment (*4,5*)).

Conventional (Time-Space Diagram) Approach

The conventional approach for determining the time and distance in queue uses a time-space (t-x) diagram to examine vehicle trajectories. Although we classify it as "conventional," this correct approach is *not* used universally in the literature, but is widely recognized. Our impetus for suggesting an alternate approach is that this conventional approach is very tedious, and has been misinterpreted in some of the literature.

The time-space diagram of Figure 1*a* represents the trajectories of vehicles approaching, queueing upstream of, and departing from, a bottleneck with capacity m and speed v_m in queue. The trajectories of vehicles in queue are nearly evenly spaced, and as such, can easily be constructed. Trajectories of vehicles at free-flow speed v_f are *not* evenly spaced in this example, indicating that the approaching traffic arrives with variable headways. The dashed line between the upstream free-flow state and the queued state represents the location of the back of the queue as a function of t.

At every point along the back-of-queue trajectory, its slope is equal to the instantaneous speed at which the location of the back of the queue is moving along the roadway. This speed, v_I, is related to Δq and Δk, the changes in flow and density across the interface (i.e., from free-flow state to queued state), by the well known relation $v_I = \frac{\Delta q}{\Delta k}$. (This relation is the basis of the kinematic wave theory of traffic flow (*6,7*).) According to this theory, the back of the queue trajectory can be constructed by piecing together small segments of the correct slope. Alternatively, since in this case the passage time of each vehicle through the bottleneck is known, the back-of-queue trajectory can be constructed as the locus of the intersections of the queued and free-flow trajectories of each vehicle. (This was the approach used in the construction of Figure 1*a*.) Both procedures are methodologically equivalent and also quite laborious. From a time-space diagram such as Figure 1*a*, all relevant measures can be measured directly.

For example, to calculate the total time spent in queue by all vehicles, T_Q, one would measure the area represented by the queued state, which in Figure 1*a* is the area enclosed by the dashed line, and multiply by the density. To calculate the total distance traveled in queue by all vehicles, D_Q, one would multiply this same area by the flow, m. This approach is tedious, however, because it involves the construction of the t-x diagram.

Proposed Approach

Consider now the trajectory of vehicle N (shown in bold in Figure 1*a*), which must queue before reaching the bottleneck. Since the bold dashed line represents the vehicle's "desired" or free-flow trajectory (once it reaches the back of the queue), the horizontal separation at the bottleneck between this desired trajectory and the actual trajectory represents, by definition, the delay, and is denoted w. For clarity, the time-space diagram of Figure 1*b* replots only the actual and desired trajectories of vehicle N, from the time and place at which the vehicle reaches the back of the queue (point BOQ), beyond the time and place it reaches the bottleneck. The vehicle's actual trajectory within the queue has a speed (i.e., slope) of v_m as shown, whereas the desired trajectory's speed is v_f. The delay w, the time spent in queue t_Q ($> w$) and the distance traveled in queue d_Q are clearly shown on the figure, and the relationship among these three variables is discussed below.

From the geometry of Figure 1*b*, it is clear that w varies with d_Q as follows:

$$w = \left(\frac{1}{v_m} - \frac{1}{v_f} \right) d_Q, \tag{1}$$

and thus

$$d_Q = \frac{w}{\frac{1}{v_m} - \frac{1}{v_f}}. \tag{2}$$

Therefore, the distance traveled in queue by a particular vehicle only depends on the arrival pattern of vehicles as it affects the vehicle's delay. Because the speeds are fixed for a given facility discharging at a given capacity m, the distance traveled in queue is a fixed multiple of the delay w. Since $t_Q = \frac{d_Q}{v_m}$, the time spent in queue is

$$t_Q = \frac{w}{1 - \frac{v_m}{v_f}}, \tag{3}$$

which is again a fixed multiple of w.

Because of this relationship between t_Q and w for an individual vehicle, the total time spent in queue by all vehicles, T_Q, will be equal to the total delay W (computed by summing all of the individual vehicle delays) multiplied by that same constant multiple, i.e.:

$$T_Q = \frac{W}{1 - \frac{v_m}{v_f}}. \tag{4}$$

Likewise, the total distance traveled by all vehicles in queue, D_Q, is

$$D_Q = T_Q \cdot v_m = \frac{W}{\frac{1}{v_m} - \frac{1}{v_f}}. \tag{5}$$

What is appealing about Equations 2, 3, 4 and 5 is that d_Q, t_Q, T_Q and D_Q have been related to the qualities w and W that can be estimated without the laborious *t-x* construction. The standard approach for estimating w and W is to use a typical input-output diagram (Figure 2; see, e.g., (*8*)), as described below.

The proposed approach first requires the construction of a typical input-output diagram. First, the arrival time of each vehicle at an upstream observation point is measured, and plotted on the figure as the curve $A(t)$. Then, by translating the arrival time of *each* vehicle horizontally to the right by the free-flow travel time to the bottleneck, t_f, the desired (or "virtual") arrival time of each vehicle at the bottleneck can be plotted as the curve $V(t)$. Finally, the departure curve $D(t)$, defining the time that each vehicle departed the bottleneck, can then be constructed in the usual way to serve the virtual arrivals at a maximum rate m. For a given vehicle number n, the horizontal separation between $V(t)$ and $D(t)$ represents the delay for that vehicle, and is denoted w_n.

Using the relationship in Equation 3, the input-output diagram of Figure 2 can be modified to include a curve $B(t)$, the number of vehicles to reach the back of the queue by time t, or equivalently the times that each vehicle reached the back of the queue (see Figure 3). We can determine the time that each vehicle joined the back of the queue by "extending" the delay of each vehicle, w_n, to the left by the factor in Equation 3. The locus of these points for all vehicles represents the "back of queue" curve, $B(t)$, which can now be constructed on the input-output diagram (Figure 3). Obviously, $B(t)$ will differ from $V(t)$ only for those vehicles for which $V(t)$ differs from $D(t)$; i.e., whenever a queue is present.

In addition to showing the t_Q for individual vehicles, the $B(t)$ curve in this "modified" input-output diagram conveniently displays many other measures. Figure 3 shows the number of vehicles in queue at time t as the vertical separation between the $B(t)$ and $D(t)$ curves. It must be remembered that the vertical separation between $V(t)$ and $D(t)$ represents the vehicles in an imaginary (point) queue where the only vehicles included are those for which their desired free-flow departure time from the bottleneck has expired. Obviously physical queues of traffic upstream of a bottleneck contain both this set of vehicles and a set of vehicles for which their desired free-flow departure time from the bottleneck has not *yet* expired, but will do so before they clear the bottleneck.

It should be obvious that the vehicle which joins the queue at its maximum length will experience the longest time in queue (and hence the greatest delay). Because the queue length is the vertical separation between $B(t)$ and $D(t)$, and the time in queue is the horizontal separation between $B(t)$ and $D(t)$, these two maximum values, $Q^{\max}$ and $t_Q^{\max}$, are related algebraically as $Q^{\max} = m\ t_Q^{\max}$, as shown in Figure 3. The maximum queue length, $Q^{\max}$, has units of vehicles; this can be converted to a physical distance as follows:

$$d_Q^{\max} = \frac{Q^{\max} \cdot v_m}{m} = t_Q^{\max} \cdot v_m. \tag{6}$$

Note from the figure that if we translate $D(t)$ vertically until it is tangent to $B(t)$ we obtain the time when the maximum queue occurs.

The total time spent by all vehicles in queue, T_Q, is the sum of each individual vehicle's t_Q, and therefore is represented graphically as the area between the curves $B(t)$ and $D(t)$ (just as the total delay of all vehicles is the area between the $V(t)$ and $D(t)$ curves). The total distance traveled in queue by all vehicles, D_Q, is therefore the product of this area and v_m. Alternately, if the horizontal axis of Figure 3 is rescaled (i.e., multiplied by v_m), the area between the $B(t)$ and $D(t)$ curves can directly display the total distance traveled by all vehicles in queue, as shown by the second horizontal axis.

The proposed approach is simple enough that it can easily be incorporated into a spreadsheet. All of the measures displayed by the modified input-output diagram (Figure 3) can be generated automatically if one is given the arrival time A_n of each vehicle n at the upstream observer, the bottleneck capacity m, the speeds v_f and v_m, and the free-flow travel time to the bottleneck t_f, as described below. (Automation of the proposed approach will be most useful when individual vehicle arrival times are known. When individual arrival times are estimated from aggregate data, one can approximate the A_n by interpolation.)

For each vehicle n, the virtual arrival time at the bottleneck V_n can be calculated as $V_n = A_n + t_f$, where t_f is the free-flow trip time from the observation point to the bottleneck. The departure time D_n of each vehicle (with the exception of the first, which we assume proceeds with no delay, so $D_1 = V_1$) can be calculated as $D_n = \max(V_n, D_{n-1} + \frac{1}{m})$, yielding a vehicle's delay as $w_n = D_n - V_n$. Given the delay of each vehicle w_n, the t_Q and d_Q are given by Equations 3 and 2 respectively. T_Q and D_Q can then be calculated by summing across individual vehicles. The maximum physical queue length is simply the maximum distance traveled by a vehicle; the time at which this maximum queue occurs is the time that vehicle reached the back of the queue. Finally, the number of vehicles in queue at time t can be determined discretely as the product of every vehicle's distance in queue, $d_{Q,n}$, and the density of vehicles in queue, $k_m = \frac{m}{v_m}$.

DEPARTURE RATE WHICH CHANGES ONCE

We believe that the approach described in the previous section can be extended to time-dependent bottlenecks. This is a difficult problem that can only be solved easily (without the t-x diagram) in the special case where the relationship (the fundamental diagram) between m (or q) and k_m on the homogeneous section upstream of the bottleneck is triangular (as in Figure 4), and with some additional difficulty if it is concave (*4*). While the general case of time-dependent bottleneck capacities under concave q-k relationships is still being investigated, this section will show how the proposed approach can be extended to the special case where the departure rate changes once at a known time and where the q-k relationship is triangular.

By dealing with a single change in bottleneck capacity, certain thorny problems are avoided, and the solution can be described relatively easily. However, a change in the bottleneck capacity implies that the departure rate that will be in place at the bottleneck when a particular vehicle departs is not necessarily known when that vehicle enters the queue. This introduces the added complication that the trip time of a vehicle is not readily known, since its velocity might change from one queued state to the other.

The triangular q-k relationship has been supported by empirical evidence (*9,10,11*), and is a reasonable first approximation. (The triangular relationship can be defined by means of only three simple quantities: the free-flow speed v_f, the maximum flow q_{max} and the jam density k_j.)

This section proceeds by first demonstrating the graphical procedure for this specific case. It then continues by describing the limiting cases of the proposed technique. Finally, the diagram produced by the graphical procedure is interpreted.

Diagram Construction

Suppose that the bottleneck can discharge at a maximum rate m_1 until time t_J, and at a maximum rate m_2 thereafter, as shown by the piece-wise linear departure curve, $D(t)$, in Figure 5. This curve can be constructed in the usual way from any given "virtual" arrival curve, $V(t)$, such as the one shown in the figure. To construct the $B(t)$ curve, we must recognize that there exist three "types" of vehicles, which are distinguished by the characteristics of the queue when the vehicle joins it, and by the bottleneck departure rate when the vehicle passes the bottleneck.

The first group of vehicles enter the queue and clear the bottleneck before the bottleneck changes capacity, and are therefore unaware that the bottleneck capacity will eventually change. This group is shown on Figure 5 as vehicles n_L to n_J; these vehicles experience only queued state 1 and can be treated as in the previous section describing the constant departure rate case.

The second group of vehicles are those which experience both states within the queue; this group is shown in Figure 5 as vehicles n_{J+1} to n_K. These vehicles join the queue at a time before the "information" about the change in the bottleneck departure rate has had time to reach the back of the queue. Thus, they join the queue in state 1 but leave in state 2. These vehicles must join the queue at the same time they would have joined it had the departure rate not changed. Therefore, to construct the back of queue curve for these vehicles, it is simplest to "imagine" that the discharge rate never changes, and to continue the procedure for the type 1 vehicles using an extrapolated departure curve, $D_1'(t)$. Before describing how the last vehicle in this group, n_K, is identified it is convenient to study the third group of vehicles.

These vehicles—shown in Figure 5 as vehicles n_{K+1} to n_M —queue only in state 2. Because the entire time in queue is spent in state 2, there is no longer any evidence that vehicles ever departed at m_1. Therefore, only $D_2(t)$, m_2 and v_2 influence $B(t)$, and this curve can be constructed from these data as explained in the previous section describing the constant departure rate case.

In summary, two component curves $B_1(t)$ and $B_2(t)$ should be constructed from the relevant departure curves $D_1(t)$ and $D_2(t)$. Starting from the "outer" points (L and M in Figure 5) the curves are constructed "inwards" (i.e., $B_1(t)$ to the right and $B_2(t)$ to the left). If the component back of queue curves $B_1(t)$ and $B_2(t)$ intersect at only one point (the usual case), then this point identifies the last vehicle to travel in both queued states (i.e., vehicle n_K in Figure 5), and our recipe is completed. The segment of $B_1(t)$ to the *left* of this intersection point combined with the segment of $B_2(t)$ to the *right* of this point yield the desired curve, $B(t)$.

If the curves intersect at multiple points, then the relevant point can be identified from the slope of the line JK, which is the rate at which the "information" wave crosses vehicles within the queue, m_I. For the triangular relation of Figure 4 this rate is constant and known to be $q_{max} / (1 - q_{max}/k_j v_f)$ (*4*).

Limiting Cases

The proposed technique breaks down for $m = 0$ or $m = q_{max}$ because the ratio in Equation 3 is not defined in these cases. Thus, alternate procedures must be developed for these cases; e.g., by examining the limiting behavior of the method as the capacity of the bottleneck moves arbitrarily close to zero or q_{max}. This will be accomplished by relating $B(t)$ to $V(t)$, instead of relating it to $D(t)$, as explained below.

First note from Equation 3 that the additional time that a vehicle N spends in queue, Δ_N ($\equiv t_{Q,N} - w_N$), is related to the delay, w_N, by:

$$\Delta_N = w_N \left(\frac{v_m}{v_f - v_m} \right). \tag{7}$$

This quantity, which represents the horizontal separation between $V(t)$ and $B(t)$ for vehicle N, turns out to be well-defined for $m \to 0$ and $m \to q_{max}$. For very small m (i.e., very small v_m) Equation 7 reduces to $\Delta_N \approx w_N \frac{v_m}{v_f}$. Additionally, since for very small v_m, $w_N \approx \frac{N}{v_m k_j}$, we find:

$$\Delta_N \approx \frac{N}{v_f k_j}, \quad \text{for } m \to 0. \tag{8}$$

From Equation 7, it is also clear that, as $m \to q_{max}$ (i.e., $v_m \to v_f$), Δ_N becomes extremely large, and therefore the back of queue curve must approach a horizontal line. The next section will demonstrate that these limiting procedures are applicable to an undersaturated traffic signal, where the capacities during the red and green phases, m_R and m_G, are 0 and q_{max}, respectively.

Interpretation of the Diagram

The straight line *JK* in Figure 5 indicates the time when the "information" concerning the change in capacity at the bottleneck reaches vehicles, and therefore identifies the time that each of these vehicles changes from queued state 1 to queued state 2. (Analogously, *JK* represents the cumulative number of vehicles to have changed state as a function of time, which is exactly the flow of vehicles that would be seen by a moving observer traveling backwards with this "information" wave.) Line *JK* must have a constant slope, because, from the time the bottleneck capacity changes, this information travels backwards towards the end of the queue at a constant velocity, and vehicles in queue are assumed to be evenly spaced. Because this line distinguishes travel in the two queued states, it conveniently permits a graphical interpretation of the total time spent by all vehicles in each state: the area between the curves $B(t)$ and $D_1(t)$ to the *left* of line *JK* (area *JKL*) represents the total time spent in state 1, and the area between the curves $B(t)$ and $D_2(t)$ to the *right* of the line (area *JKM*) represents the total time spent in state 2. The total distance traveled by all vehicles in each of the queued states is simply the product of the total time spent in the queued state and the speed in that queued state.

As for the constant capacity case, the total distance traveled in each queued state can also be interpreted graphically. For clarity, the modified input-output diagram of Figure 5 is redrawn as Figure 6, and includes two additional "distance" axes, which have been rescaled from the time axis by the two queued speeds, v_1 and v_2, respectively. The total distances traveled in queued states 1 and 2 are the areas *JKL* and *JKM*, respectively, where distance axis 1 is used for *JKL* and axis 2 for *JKM*.

The distance traveled in queue by a vehicle which travels in a single queued state can be read directly by using the appropriate axis. For vehicles traveling in both queued states, distances in state 2 increase linearly with vehicle number. As a result, it is possible to introduce an auxiliary straight line, $D^*(t)$, that will allow us to read the combined distance from distance axis 1 as the horizontal separation between $B(t)$ and $D^*(t)$. As shown in the figure, the horizontal separation of this line from line *JK* must be $\frac{v_2}{v_1}$ times larger than the horizontal separation between *JK* and *JM*.

The time and distance in queue of individual vehicles which travel in both queued states can also be obtained numerically by considering the trajectory of a single vehicle *N* in Figure 5. It can be shown that the time that vehicle *N* spends in state 2, $t_{N,2}$, can be expressed numerically as:

$$t_{N,2} = \left(\frac{1}{m_2} - \frac{1}{m_I} \right) \left(N - n_J \right). \tag{9}$$

The time vehicle *N* spends in queued state 1 can be found by substituting that vehicle's delay in queued state 1, $w_{N,1}$, into Equation 3, where $w_{N,1}$ can be calculated as follows:

$$w_{N,1} = w_N - t_{N,2} \left(1 - \frac{v_2}{v_f} \right). \tag{10}$$

Given the times in each queued state, the distances traveled by a vehicle can be calculated by multiplying these times by their appropriate queued speed.

As in the single capacity case, Figure 5 shows the number of vehicles in queue at time t as the vertical separation between the $B(t)$ and $D(t)$ curves. It should be noted that, although the maximum physical queue length can still be determined as the maximum distance that any vehicle spends in queue, this value cannot necessarily be determined using Equation 6, because some vehicles travel two different speeds in queue.

Again, the simplicity of the proposed approach in this more general case allows the technique to be automated in a spreadsheet. Once the vehicles represented by points *J* and *K* in Figure 5 are identified, the appropriate equations can be used to provide the time and distance that each vehicle spends in each of the queued states. Many of the other measures can then be generated as described in the previous section.

UNDERSATURATED TRAFFIC SIGNAL EXAMPLE

The behavior of traffic upstream of an undersaturated traffic signal (i.e., where the queue clears during the green phase) with a constant arrival rate, l, is now examined with the proposed procedure. This particular case is examined because it is a well-known problem which will allow us to compare the results of the proposed procedure with the conventional ones. Consider once again the *q-k* relationship of Figure 4. In the case of an undersaturated traffic signal placed on a homogeneous road, the capacities during the red and green phases, m_R and m_G, are $m_R = 0$ and $m_G = q_{max}$. The corresponding speeds, v_R and v_G, are $v_R = 0$ and $v_G = v_f$.

The conventional time-space diagram for this problem is given in Figure 7. Since $v_R = 0$ and $v_G = v_f$, we see from the figure that the delay to a vehicle is its time in state 1.

The proposed approach yields the component curves, $B_1(t)$ and $B_2(t)$, of Figure 8. As explained in connection with the limiting cases for $m = 0$ and $m = q_{max}$, curve $B_2(t)$ is horizontal and curve $B_1(t)$ diverges from $V(t)$ at a constant rate (Equation 8) with increasing vehicle number. Recall, as well, that the straight line *JK* in Figure 8 represents the time at which a vehicle would change from the first to the second queued state.

Careful inspection of both the time-space and input-output diagrams reveals that they are perfectly consistent (although Figure 8 is easier to construct and interpret, especially in a case where the arrival rate would depend on time). For example, vehicle n_K has zero delay but travels the greatest distance in queue; the time at which vehicle n_K reaches the back of the queue, t_K, is thus the time when the queue has its greatest physical length. (It is interesting to note that the maximum physical queue length is longer than would be predicted assuming "point" queueing, and the time at which this maximum occurs is later than would have been predicted with point queues, t_J.)

CONCLUSIONS

This paper has presented a simple approach for determining the spatial and temporal extents of a queue upstream of a bottleneck, using a modified input-output diagram. The inherent advantages of the basic input-output diagram—its simplicity and utility—make it quite useful in its traditional form for measuring the delay within a system. The modification to the input-output diagram proposed in the paper (adding the "back of queue" curve) makes this tool even more versatile, enabling the evaluation of waiting times, distances traveled in queue, and queue lengths without the laborious construction of the more conventional time-space diagram.

The proposed graphical approach should be particularly useful, because one can see at a glance how measures of performance depend on decision variables and specific data. These measures allow a proper examination of several impacts of various traffic control and design alternatives, including fuel consumption and emissions. The graphical approach further indicates the maximum queue length, and the time at which this maximum occurs, quite clearly. These measures, which can easily be miscalculated using other means, have quite significant implications on the storage requirements at intersections, and the coordination of closely spaced traffic signals, among other things.

If one's primary goal is to determine the total time spent, or distance traveled, by all vehicles in queue, then one need not construct the "back of queue" curve as described in the paper. From a limited amount of data the simple expressions developed above can be easily automated in a spreadsheet to provide the desired result. Alternatively, these two measures can be estimated as fixed multiples of the total *delay* to all vehicles, a parameter which is calculated by many modeling packages.

Although the approach has been shown to apply under many circumstances, the technique is not applicable in its present form to the oversaturated traffic signal case, nor a bottleneck which changes capacity more than once.

ACKNOWLEDGMENT

The authors wish to thank Professor Michael Cassidy, University of California, and three anonymous reviewers for their comments and suggestions.

REFERENCES

1. Teply, S. (1989) Accuracy of Delay Surveys at Signalized Intersections. In *Transportation Research Record 1225*, TRB, National Research Council, Washington, D.C., pp. 24-32.
2. Son, Bongsoo. (1996). *A Study of G.F. Newell's "Simplified Theory of Kinematic Waves in Highway Traffic."* Ph.D. Thesis, Graduate Department of Civil Engineering, University of Toronto.
3. Dale, C.W. (1980) *Procedure for Estimating Highway User Costs, Fuel Consumption and Air Pollution*. Office of Traffic Operations, Federal Highway Administration, Washington, D.C.
4. Newell, G.F. (1993) A Simplified Theory of Kinematic Waves in Highway Traffic. *Transportation Research*, Vol. 27B, No. 4, pp. 281-313.
5. Vaughan, R. and V.F. Hurdle. (1992) A Theory of Traffic Flow For Congested Conditions on Urban Arterial Streets, I: Theoretical Development. *Transportation Research*, Vol. 26B, No. 3, pp. 381-386.
6. Lighthill, M.J. and G.B. Whitham. (1955) On Kinematic Waves II, A Theory of Traffic Flow on Long Crowded Roads. *Proceedings of the Royal Society*, Vol. A 229, pp. 317-345.
7. Richards, P.I. (1956) Shock Waves on the Highway. *Operations Research*. Vol. 4, pp. 42-51.
8. Newell, G.F. (1982) *Applications of Queueing Theory*. Second Edition. Chapman and Hall, New York.
9. Hall, F.L., B.L. Allen and M.A. Gunter. (1986) Empirical Analysis of Freeway Flow-Density Relationships. *Transportation Research*, Vol. 20A, No. 3, pp. 197-210.
10. Banks, J.H. (1989) Freeway Speed-Flow-Concentration Relationships: More Evidence and Interpretations. In *Transportation Research Record 1225*, TRB, National Research Council, Washington, D.C., pp. 53-60.
11. Cassidy, M. (1997) Bivariate Relations in Nearly Stationary Highway Traffic. *Transportation Research* (in press).

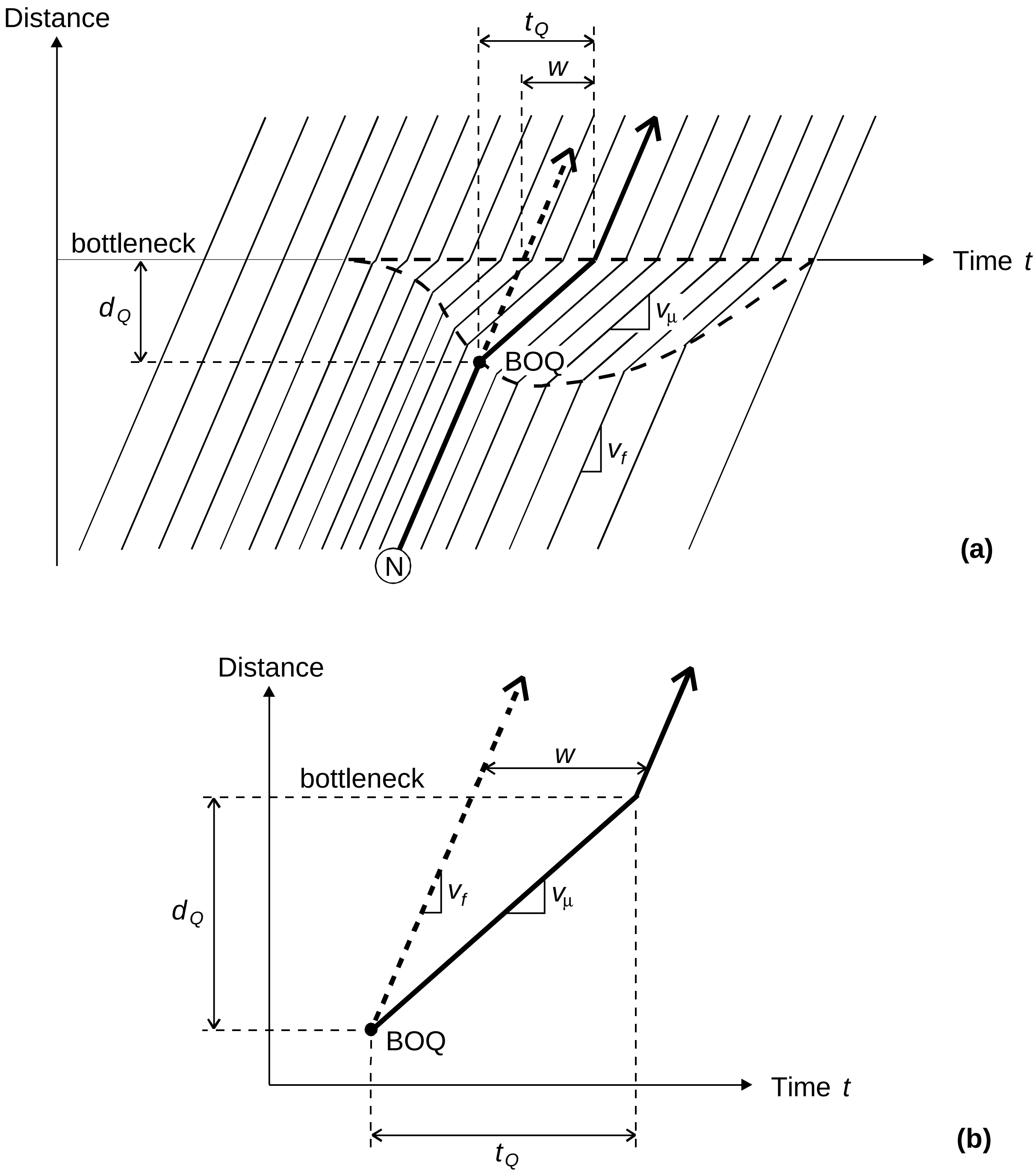

FIGURE 1 Conventional (Time-Space Diagram) Approach:
(a) Vehicle Trajectories
(b) Actual and "Desired" Trajectories of Vehicle *N*

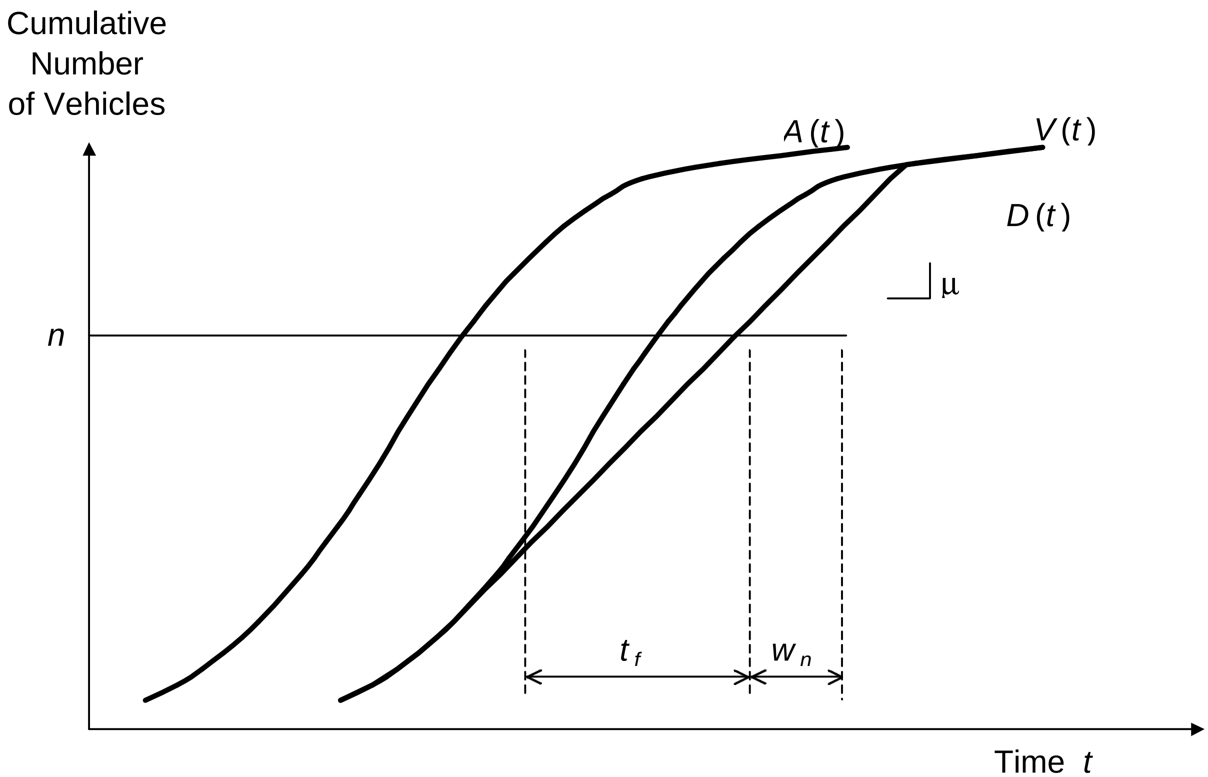

FIGURE 2 Input-Output Diagram

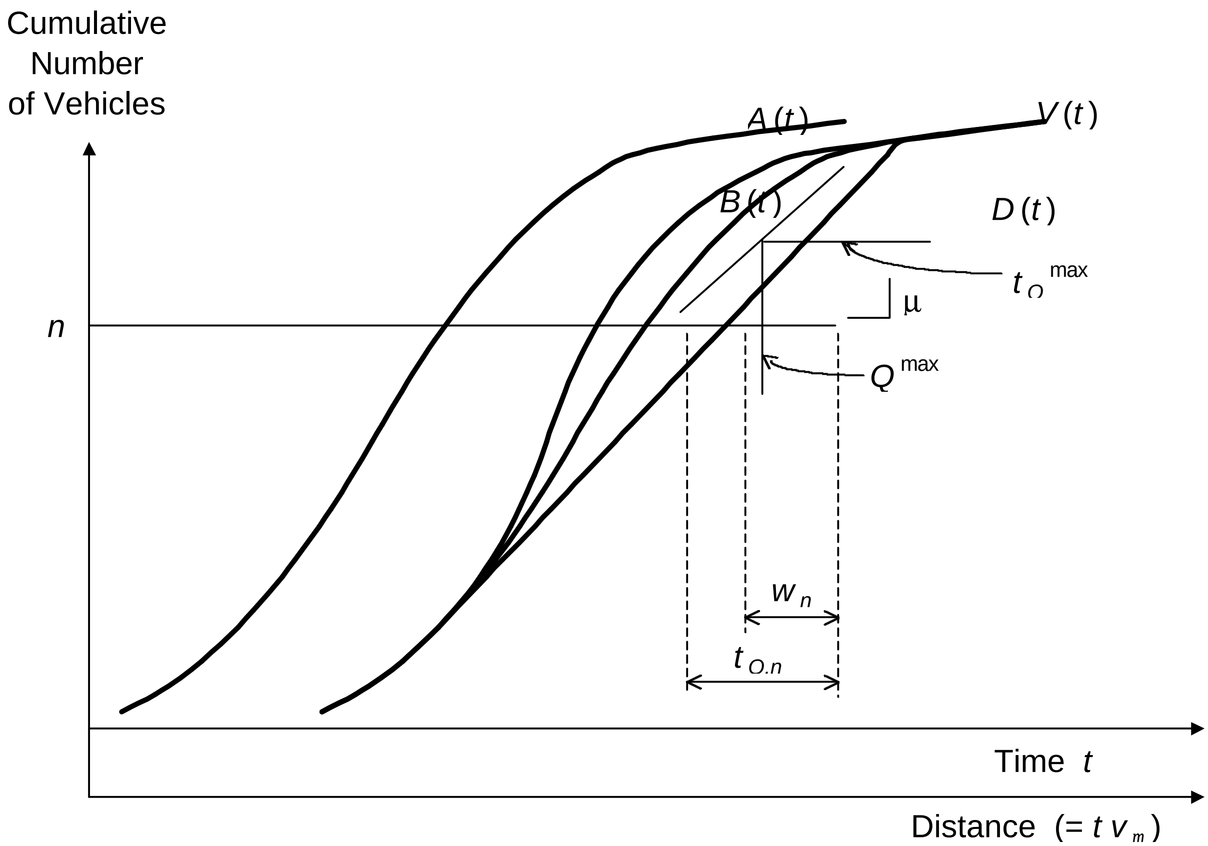

FIGURE 3 Locating the Back of Queue Curve, *B*(*t*), for Constant Departure Rate

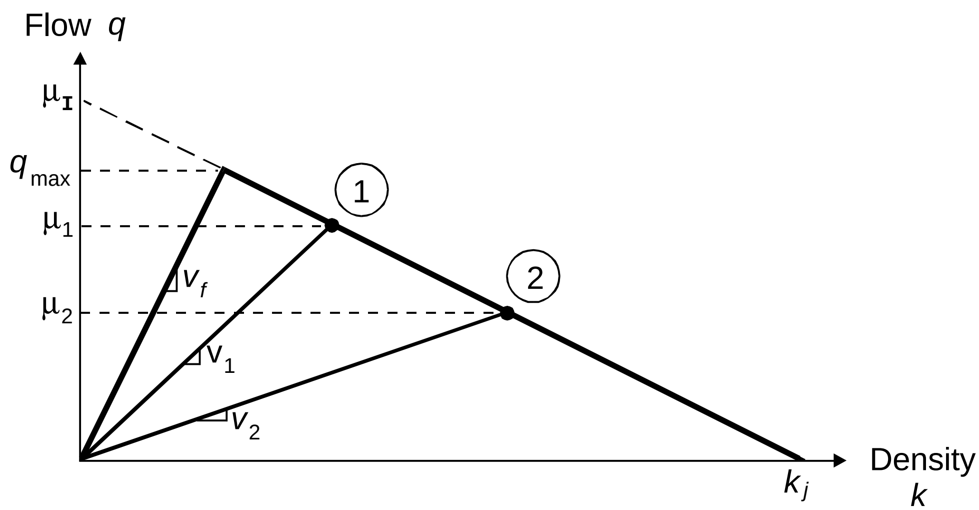

FIGURE 4   Flow Density Relationship

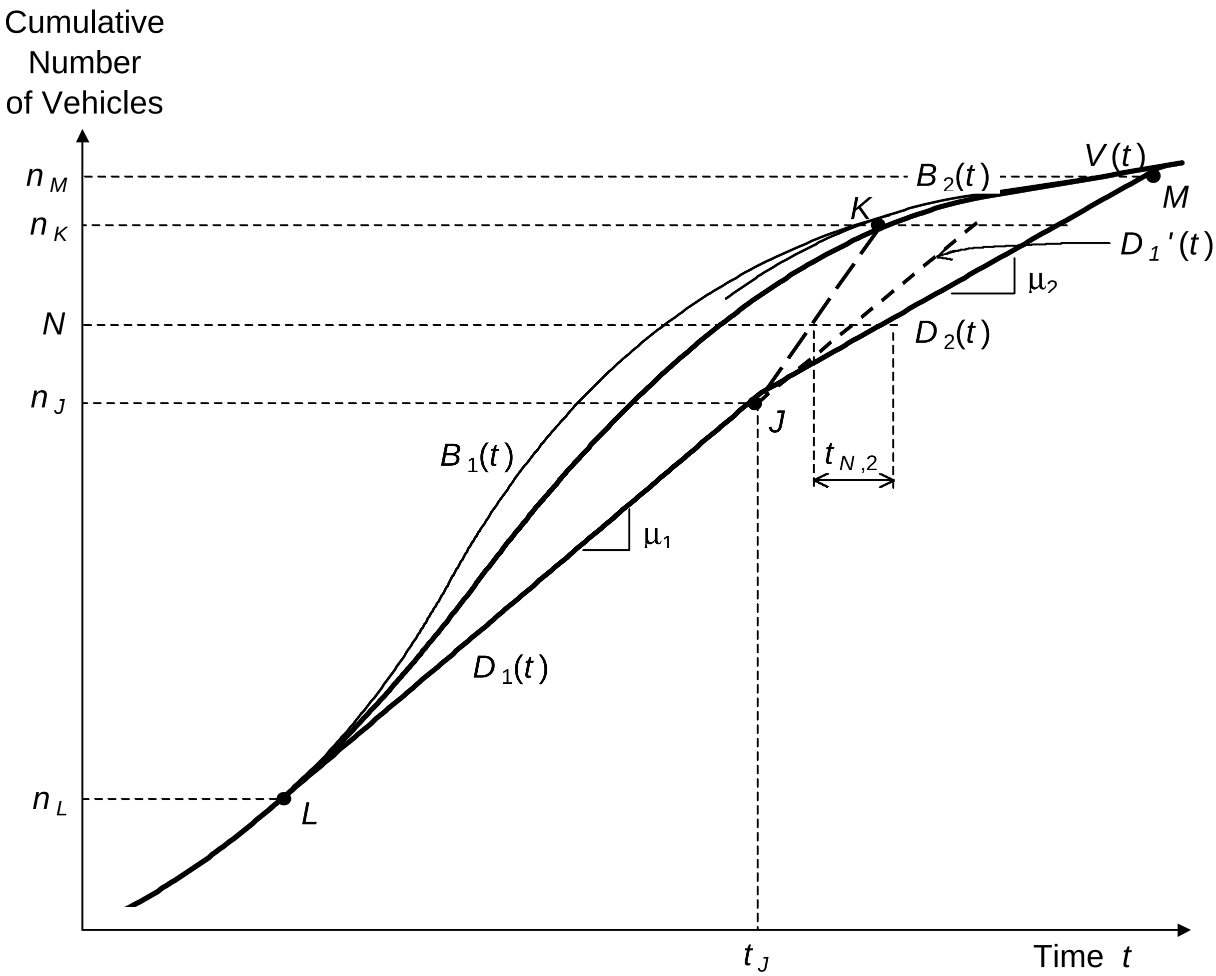

FIGURE 5 Locating the Back of Queue Curve, B(t), for Departure Rate Which Changes Once

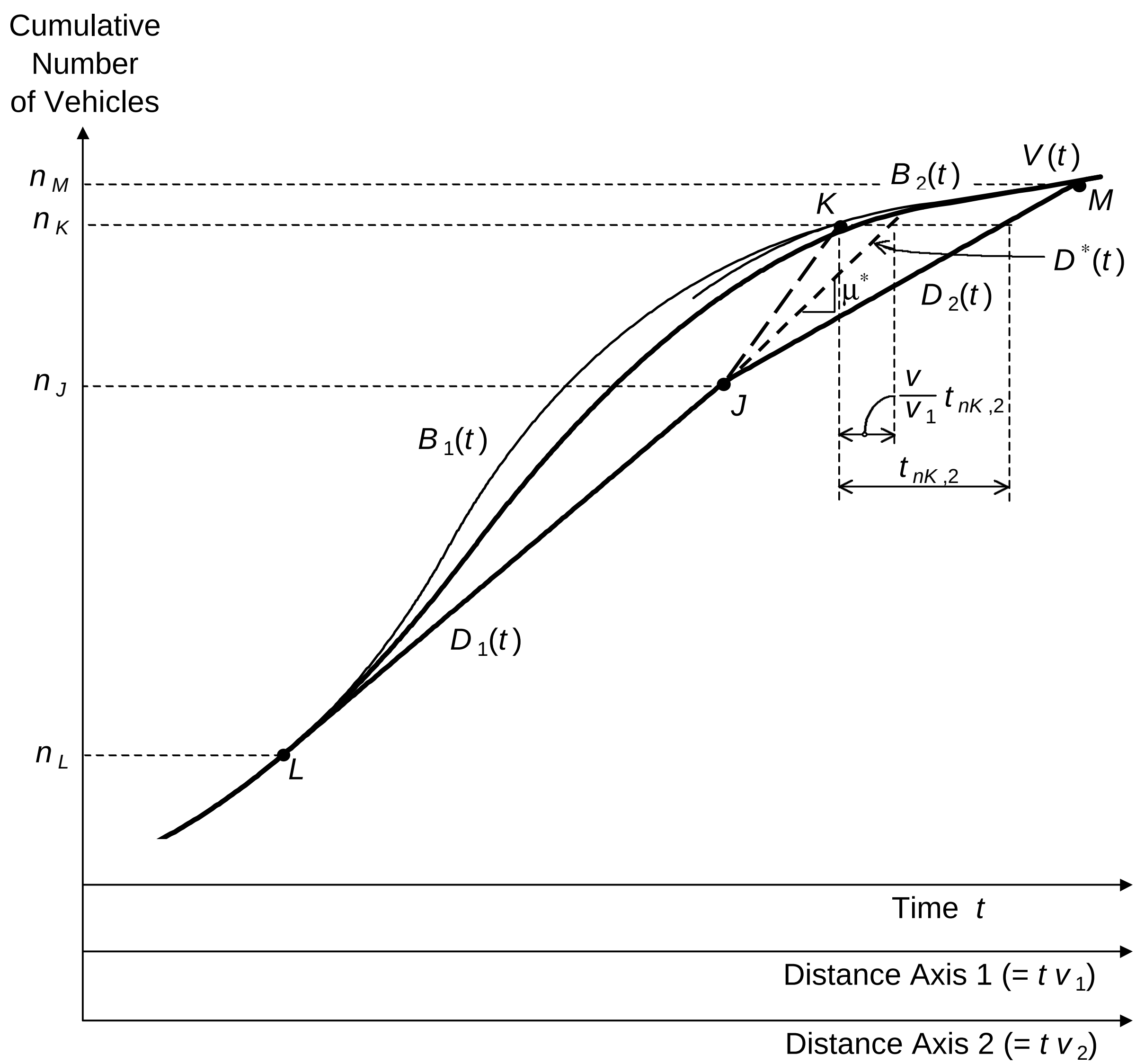

FIGURE 6 Representing Distances Travelled in Queue on the Input-Output Diagram

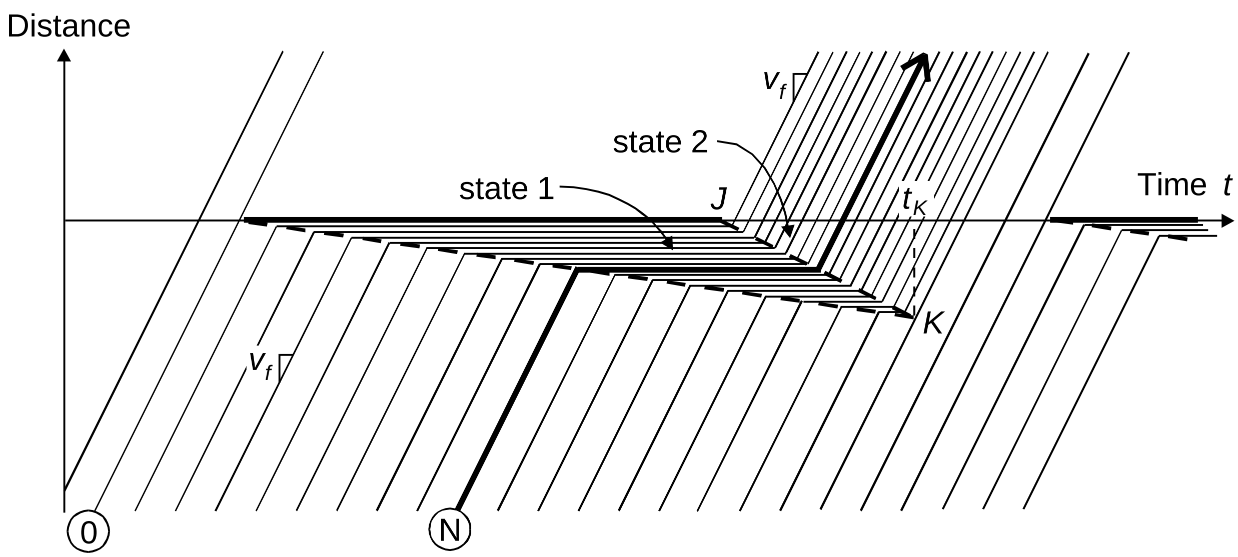

FIGURE 7 Vehicle Trajectories at the Traffic Signal

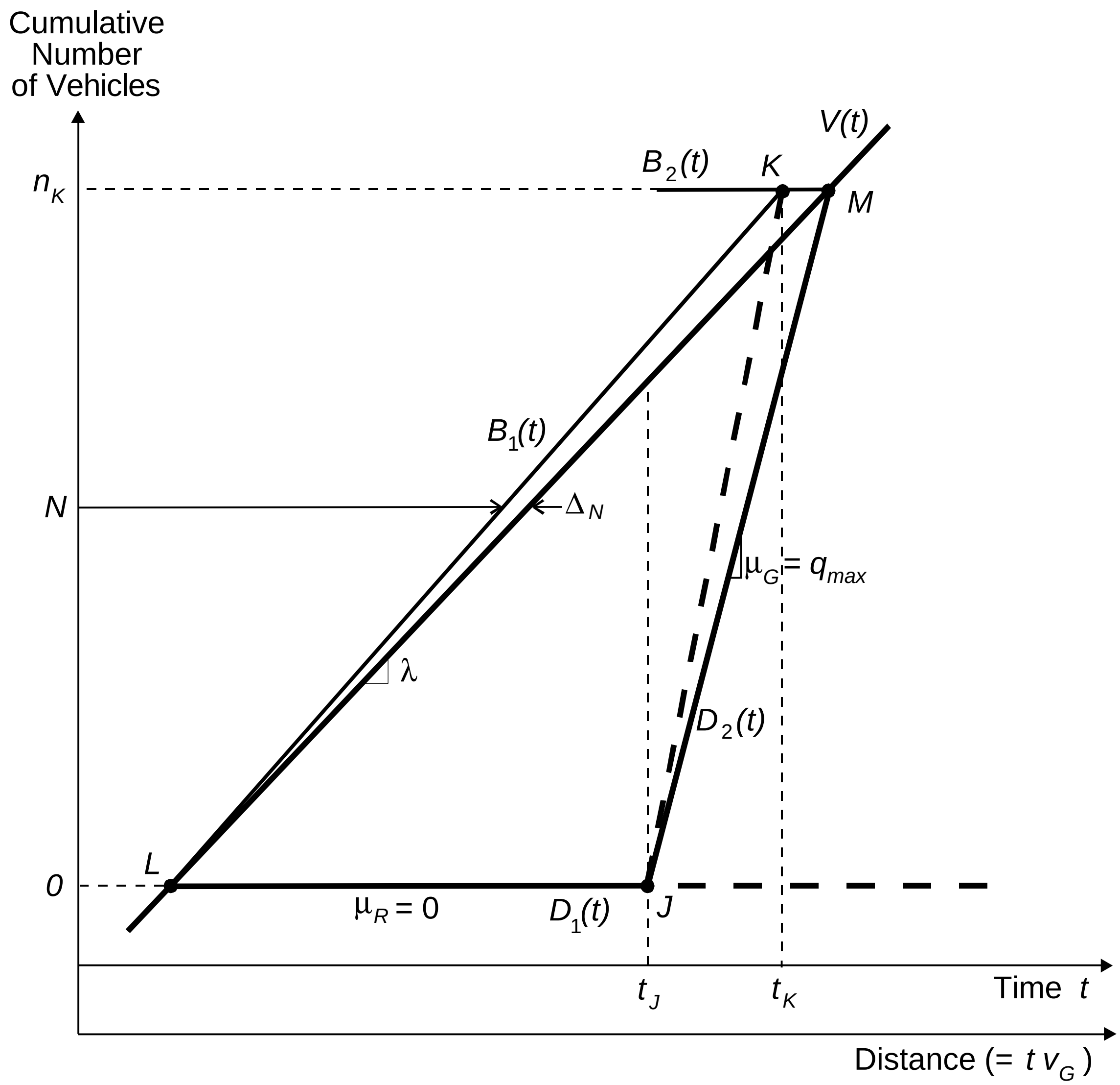

FIGURE 8 Locating the Back of Queue Curve, B(t), for the Traffic Signal